

Matlab Laboratory 4. Differential Equations in Matlab

Aim of the laboratory

- Understanding the Matlab computation engine
- Perform some mathematical operations in Matlab

Necessary equipment

- Workstations with MATLAB

Theoretical Approach

The previous laboratories have presented an overview of the Matlab computation engine, illustrating the employment of matrices, operations with matrices, program control statements and plots. Further on, this laboratory presents the employment of Matlab to solve differential equations.

Matlab provides a very powerful solver for differential equation. Accordingly, solving differential equations becomes both easy and straightforward. The syntax of the differential equation solver is

```
[T,Y] = solver(odefun,tspan,y0,options)
```

The Matlab differential equation solvers are *ode23*, *ode45*, *ode113*, *ode15s*, *ode23s*, *ode23t* and *ode23tb*. The characteristics of the different solvers can be consulted in the Matlab help.

The differential equation to be solved must be written in advance as a Matlab function, and is transmitted to the solver through function handle *odefun*. *Tspan* states the integration interval and *y0* state the initial conditions. Further options can be placed through *options*, as indicated in the Matlab help.

To illustrate the Matlab differential equation solver, consider plotting the capacitor charging and discharging voltage and current respectively of an RC, as illustrated in figure 5. The input voltage in this case is considered a square wave which switches between positive DC voltage V_{DC} and ground.

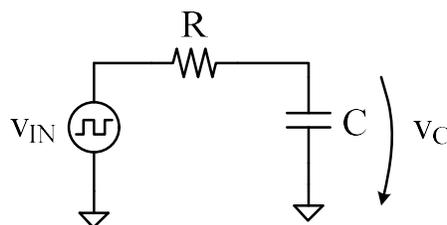


Figure 5.

Depending on the state of the input voltage, the circuit from figure 5 has two phases of operation. When the input voltage is high, the capacitance charges from DC voltage source V_{DC} through passive resistance R . The equivalent circuit is illustrated in figure 6 (a).

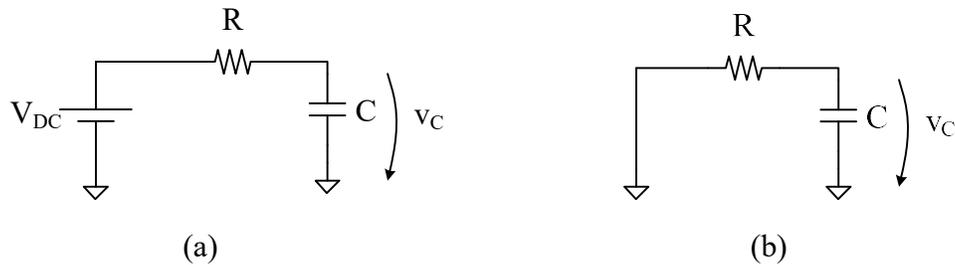


Figure 6.

Next, when the input voltage is low, the capacitance discharges to ground through passive resistance R . The equivalent circuit is illustrated in figure 6 (b).

Regardless of the phase of operation, the capacitance current is given by

$$i_C = C \cdot \frac{dv_C}{dt}$$

For the capacitance voltage, the two operation phases must be treated separately. During the charge phase, Kirchoff's law expresses that

$$V_{DC} = R \cdot i_C + v_C$$

The expression of the capacitance current in permanent regime is then expressed as

$$i_C = \frac{V_{DC} - v_C}{R}$$

which leads to the expression of the capacitance charging voltage

$$\frac{dv_C}{dt} = \frac{V_{DC} - v_C}{R \cdot C}$$

Similarly, during the discharge phase, Kirchoff's law expresses that

$$R \cdot i_C + v_C = 0$$

which leads to the expression of the discharge voltage

$$\frac{dv_C}{dt} = \frac{v_C}{R \cdot C}$$

One will observe that the capacitance current and voltage are expressed by differential equations respectively. The *solver* which is required for this exercise is **ode45**. The Matlab function which expresses the capacitance voltage during the charging phase is

```
function dy=F1(t,y,flag,VDC,R,C)
dy=(VDC-y)./(R.*C);
```

In the code section above, the key word *flag* separates the output and the input function arguments. Accordingly, the arguments before the *flag* will be returned by the function, whereas the arguments after the *flag* are required to compute the function result.

To do...

Write a second Matlab function F2 to express the capacitance voltage during the discharge phase.

Further on, the time intervals for high and low states of the input voltage, namely the capacitance charge and discharge phases respectively, must be defined. For this purpose, one must consider a rectangular control signal, such as the one illustrated in figure 7.

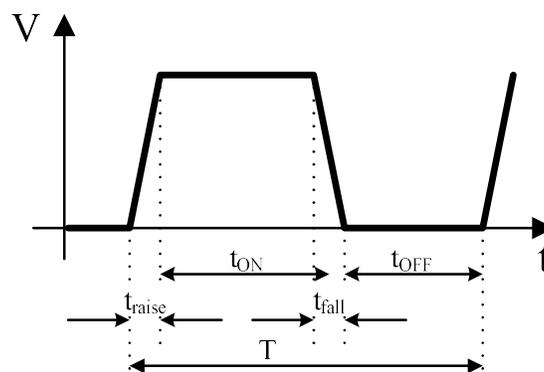


Figure 7.

One signal period (T) consists of a HIGH interval (t_{ON}) and a LOW interval (t_{OFF}). Next, the raise time (t_{raise}) and the fall time (t_{fall}) can be ideally considered zero. In reality however, a square signal is impossible as it cannot exhibit two different amplitude values in the exact same moment of time, and therefore t_{ON} and t_{OFF} exhibit finite values. Further on, the duty cycle (D) is expressed as

$$D = \frac{t_{ON}}{T}$$

To be noticed is that, while the period is given in absolute value, the duty cycle will have a sub-unitary value, with 0 standing for DC LOW state and 1 standing for DC HIGH state. To express the duty cycle in percentage, one must multiply the above equation by 100.

The function which solves the differential equations is listed as follows. To be noticed is that, besides the circuit parameters, the function also takes the number of periods to be displayed as an argument. This allows the illustration of the transient regime.

```
function RC_circuit(VDC,R,C,T,D,N)

Fig=figure('Name','Time domain behavior of RC circuits');

Tc=D.*T;
y=0; %initial conditions, capacitance is discharged
```

```

for j=1:N;

    d(j)=min(y); %initial conditions for current frame, charge phase
    [t,y]=ode45('F1',[(j-1).*T (j-1).*T+Tc],d(j),[],VDC,R,C);
    i=(VDC-y)./R; %capacitance current

    subplot(2,1,1);plot(t,y,'r'); %plot charge voltage and current
    hold on;grid on;
    subplot(2,1,2);plot(t,i,'r');
    hold on;grid on;

    a(j)=max(y); %initial conditions for current frame, discharge phase
    [t,y]=ode45('F2',[(j-1).*T+Tc j.*T],[a(j)],[],R,C);
    i=(-y)./R; %capacitance current

    subplot(2,1,1);plot(t,y,'b'); %plot discharge voltage and current
    hold on;
    subplot(2,1,2);plot(t,i,'b');
    hold on;
end;

subplot(2,1,1); %customize axes
ylabel('vc [V]');
subplot(2,1,2);
ylabel('ic [A]');
xlabel('timp [mS]');

```

To call this function, write the following Matlab script.

```

clear all;
close all;

VDC = 5;
R = 3e3;
C = 2e-6;
T = 6e-3;
D = 0.5;
N = 7;

RC_circuit(VDC,R,C,T,D,N);

```

The capacitance voltage and current are then plotted in figure 8.

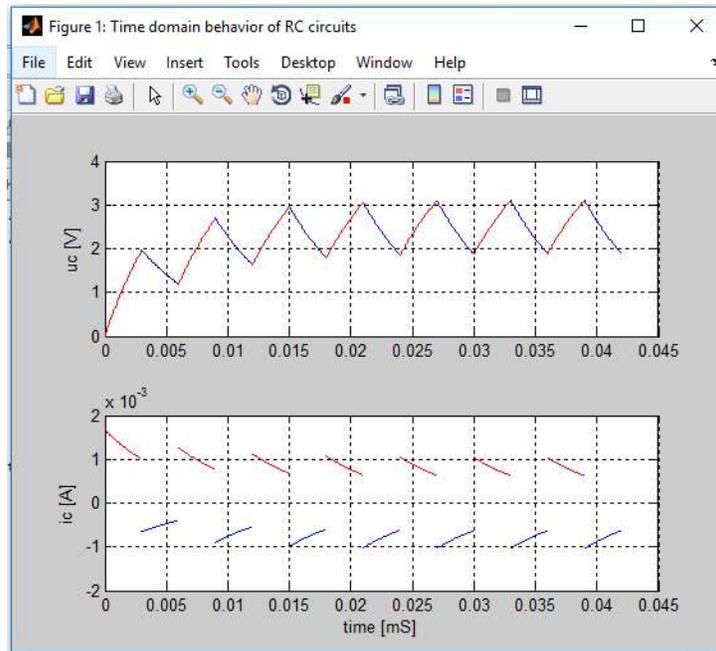


Figure 8.

Exercise 1. Plot the inductance voltage and current of the RL circuit from figure 9.

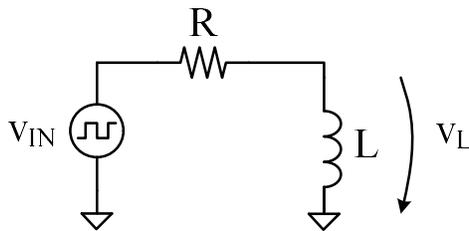


Figure 9

Depending on the state of the input voltage, the circuit from figure 5 has two phases of operation. When the input voltage is high, the inductance charges from DC voltage source V_{DC} through passive resistance R . The equivalent circuit is illustrated in figure 6 (a).

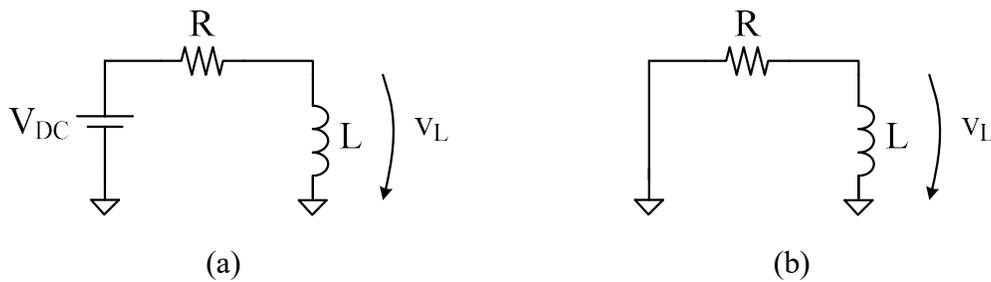


Figure 6.

Next, when the input voltage is low, the inductance discharges to ground through passive resistance R. The equivalent circuit is illustrated in figure 6 (b).

Regardless of the operation phase, the inductance voltage is given by

$$v_L = L \cdot \frac{di_L}{dt}$$

For the inductance current, the two operation phases must be treated separately. During the charge phase, Kirchoff's law expresses that

$$V_{DC} = R \cdot i_L + v_L$$

The expression of the inductance voltage in permanent regime is then expressed as

$$v_L = V_{DC} - R \cdot i_L$$

which leads to the expression of the capacitance charging current

$$\frac{di_L}{dt} = \frac{V_{DC} - R \cdot i_L}{L}$$

Similarly, during the discharge phase, Kirchoff's law expresses that

$$R \cdot i_L + v_L = 0$$

which leads to the expression of the discharge voltage

$$\frac{di_L}{dt} = -\frac{R \cdot i_L}{L}$$

Exercise 2. Astable multivibrator circuit with 555 timer

Exercise 3. Monostable multivibrator circuit with 555 timer