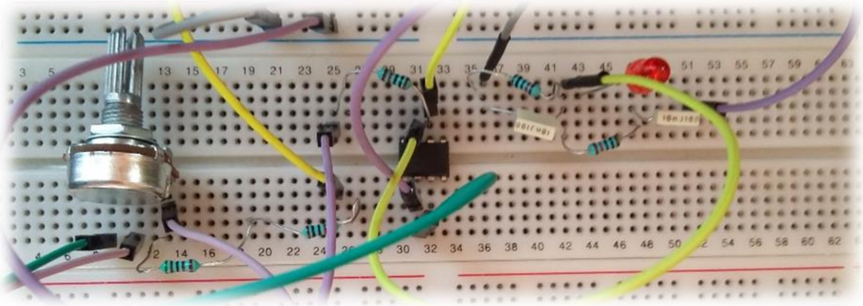


FUNDAMENTAL ELECTRONIC CIRCUITS

Assist. prof. Laura-Nicoleta IVANCIU, Ph.D.

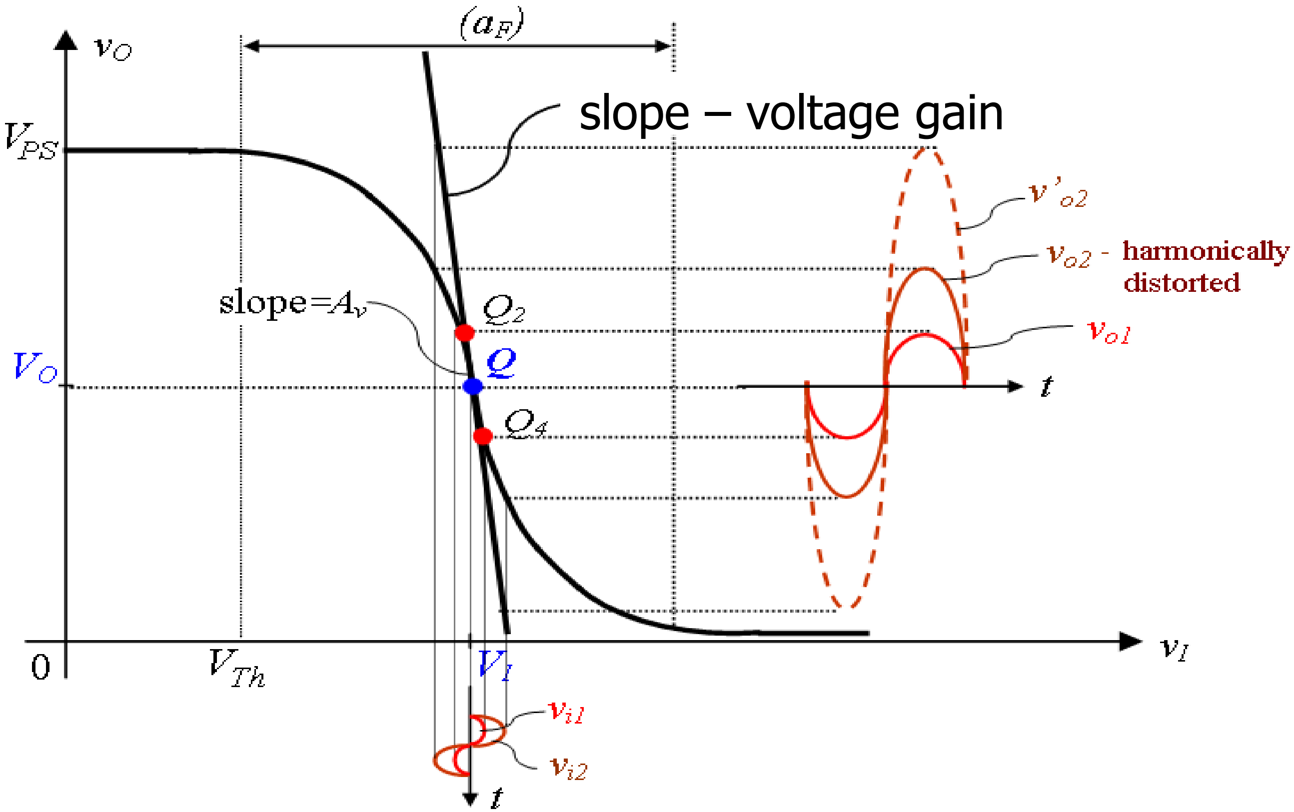
C4 – BJT basic amplifiers

Contents

- BJT small-signal model
- BJT basic amplifiers
- Comparison and analysis

BJT small-signal model

➢ Small-signal motivation - revisited



Small-signal model (linear model)

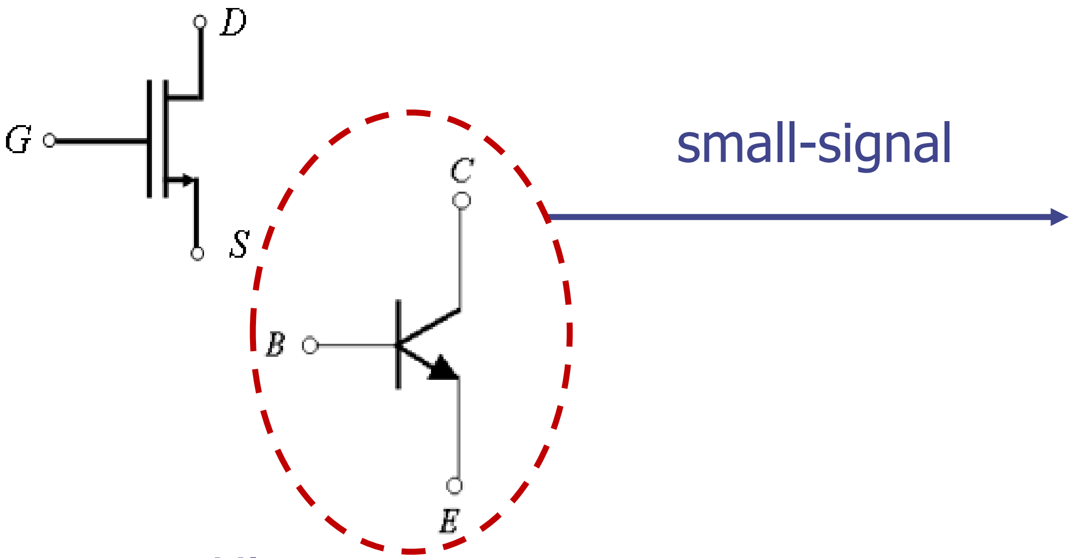
needed to compute

Voltage gain: $A_v = \dfrac{v_o}{v_i}$

Small-signal model is valid in the **narrow linear region around *Q***.

BJT small-signal model

➢ Small-signal operation - intro



Some components

Some connections

Some parameters

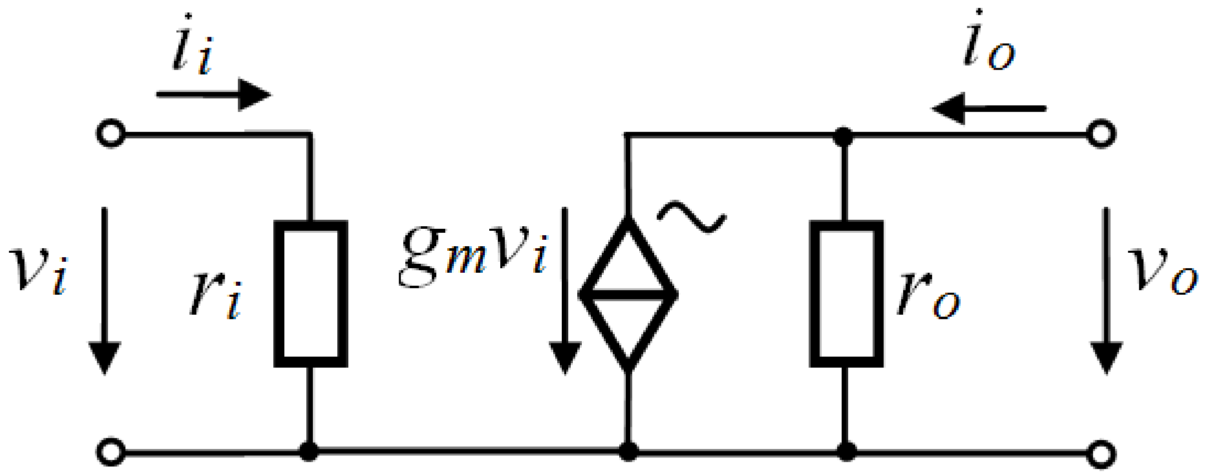
small-signal model of T

Hints:

- keep names of terminals
- use voltage-controlled current source
- values of parameters are computed in $Q\,(V_{DS};\ I_D)$ or $Q\,(V_{CE};\ I_C)$

BJT small-signal model

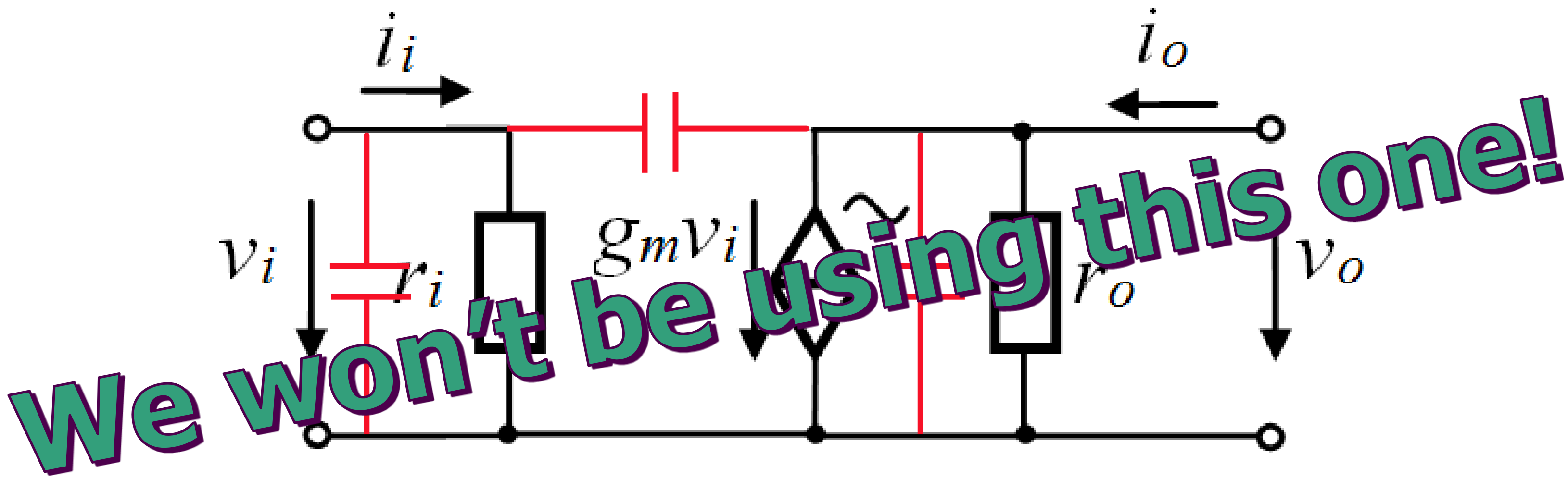
➢ Small-signal model – low & medium frequency



- **two-port network**
  - ✓ input resistance: r_i
  - ✓ transfer: voltage-controlled current source: $g_m v_i$
  - ✓ output resistance: r_o

BJT small-signal model

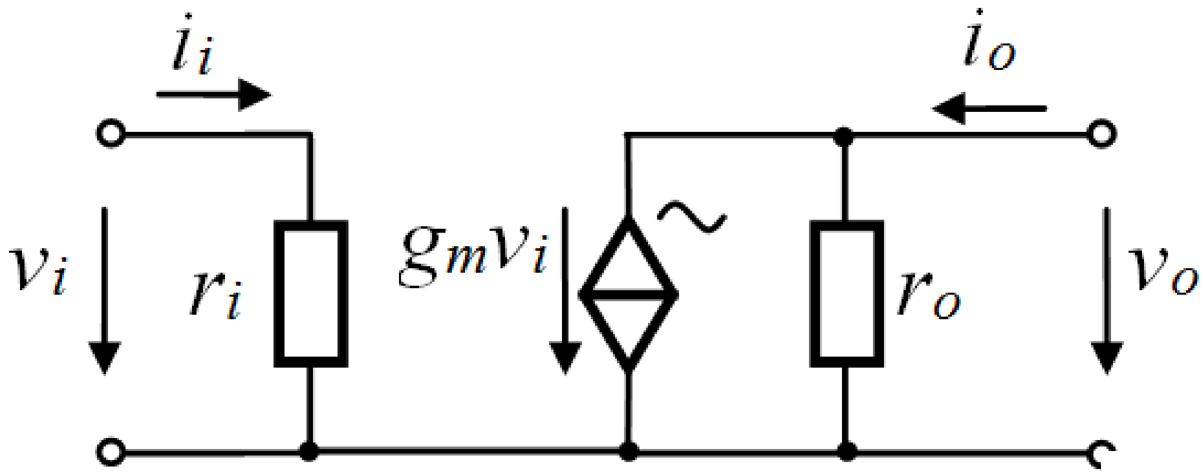
➢ Small-signal model – high frequency



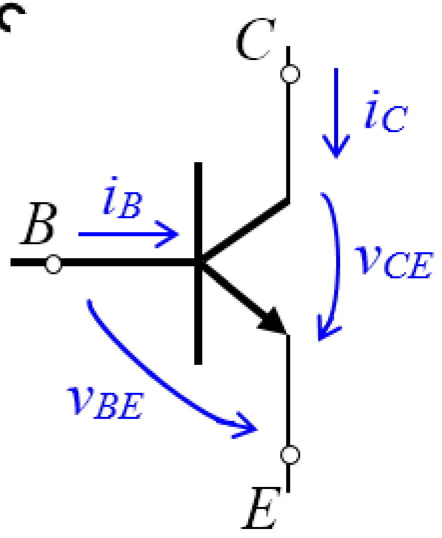
- **two-port network**
  - ✓ r_i
  - ✓ $g_m v_i$
  - ✓ r_o

+ parasitic capacitances between terminals

➢ Small-signal model – low & medium frequency



Custom model for BJT?



- ✓ $r_i = ?$
- ✓ $g_m = ?, v_i = ?$
- ✓ $r_o = ?$

BJT small-signal model

➢ Small-signal parameters

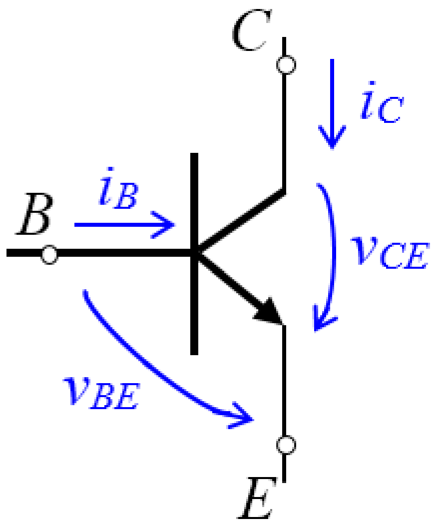
Input resistance r_i

$$r_i = r_{be} = \frac{\partial v_{BE}}{\partial i_B}\bigg|_{v_{CE}\ =\ cst} = \frac{v_{be}}{i_b}\bigg|_{v_{CE}\ =\ cst}$$

$\frac{\partial v_{BE}}{\partial i_B}$ - derivative of v_{BE} with respect to i_B

Major difference from MOSFET: $i_B \neq 0$

$$r_{be} = \frac{\beta}{g_m}$$



Who are β and g_m?

➢ Small-signal parameters

Transconductance g_m

- shows the transfer from variable input voltage v_{BE} to variable output current i_C

$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{v_{CE} = cst} = \left.\frac{i_c}{v_{be}}\right|_{v_{CE} = cst}$$

$$i_C \approx I_S e^{\frac{v_{BE}}{V_T}}$$

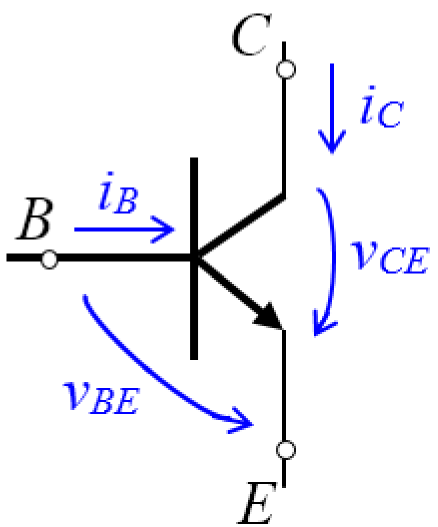
I_S - saturation current ~ nA – pA
V_T - thermal voltage, ≈ 25 mV@20ºC

$$g_m = \frac{I_C}{V_T} \approx 40 I_C \quad @20°\mathrm{C}$$

$g_m - [\mathrm{mS}]$ for $I_C - [\mathrm{mA}]$

$$V_T = \frac{KT}{q}$$

K - Boltzmann's constant
q – elementary charge (electric charge carried by a single electron)
T – absolute temperature measured in K



Temperature ↗ g_m ↘

➢ Small-signal parameters

Current gain β

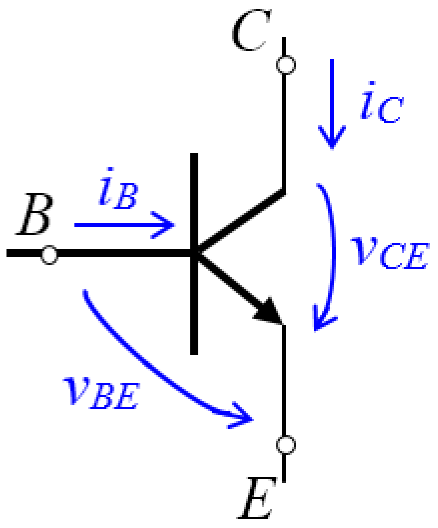
- shows the transfer from variable input current i_B to variable output current i_C

$$\beta = \left.\frac{\partial i_C}{\partial i_B}\right|_{v_{CE} = cst} = \left.\frac{i_c}{i_b}\right|_{v_{CE} = cst}$$

$\beta = 100$ - common value for BJTs

Note: there are some differences between the dc current gain and the small-signal current gain

For simplicity, we're assuming $\beta = 100$ for both dc and small-signal analysis.



BJT small-signal model

Small-signal parameters

Output resistance r_o

$$r_o = r_{ce} = \frac{1}{g_o} = \frac{\partial v_{CE}}{\partial i_C}\bigg|_{v_{BE}\ =\ cst} = \frac{v_{ce}}{i_c}\bigg|_{v_{BE}\ =\ cst}$$

$\dfrac{\partial v_{CE}}{\partial i_C}$ - derivative of v_{CE} with respect to i_C

$$r_{ce} = \frac{V_A}{I_C}$$

$$i_C = I_S e^{\frac{v_{BE}}{V_T}}\left(1 + \frac{v_{CE}}{V_A}\right)$$

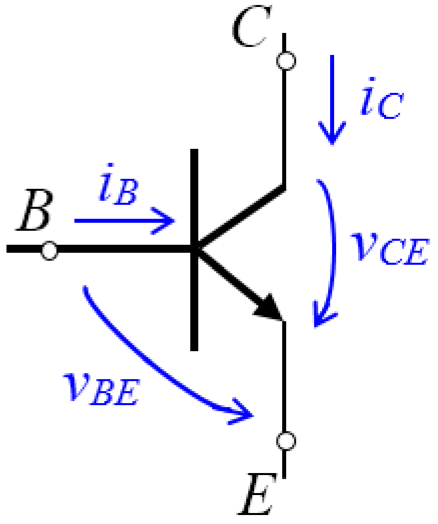
I_S - saturation current ~ nA – pA

V_T - thermal voltage, ≈ 25 mV@20ºC

V_A – Early voltage

V_A = 100 V for npn BJT

V_A = 50 V for pnp BJT



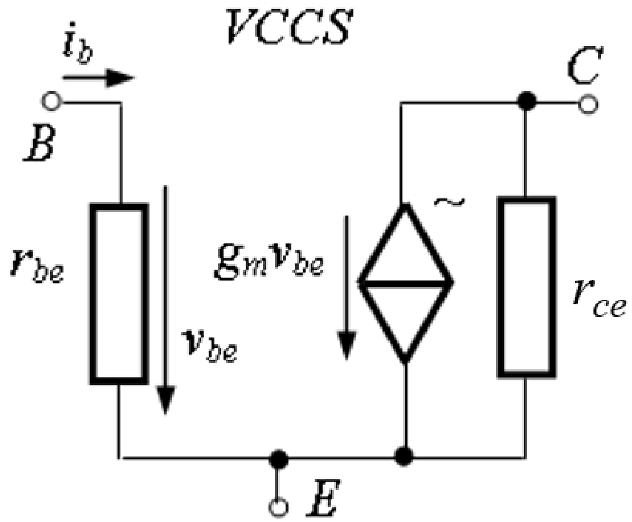
➢ Small-signal model & parameters - summary

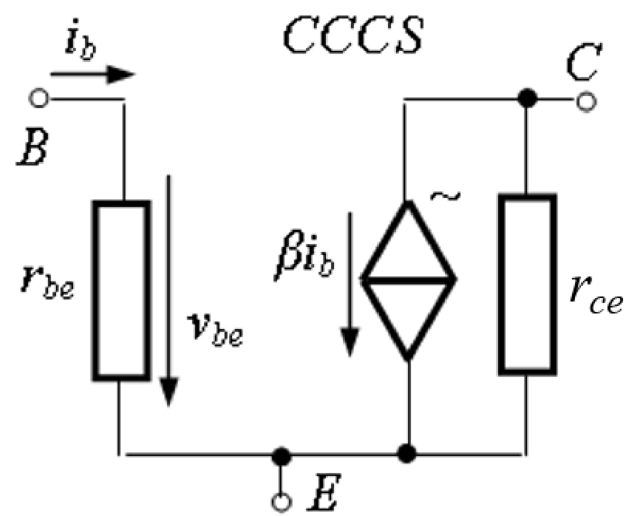
$$r_i = r_{be} = \frac{\beta}{g_m} \qquad g_m = 40I_C \qquad r_o = r_{ce} = \frac{V_A}{I_C} \qquad i_c = g_m v_{be} = 40 I_C v_{be}$$

hybrid-π models

VCCS – voltage-controlled current source

CCCS – current-controlled current source





Valid for low & medium frequency

BJT small-signal model

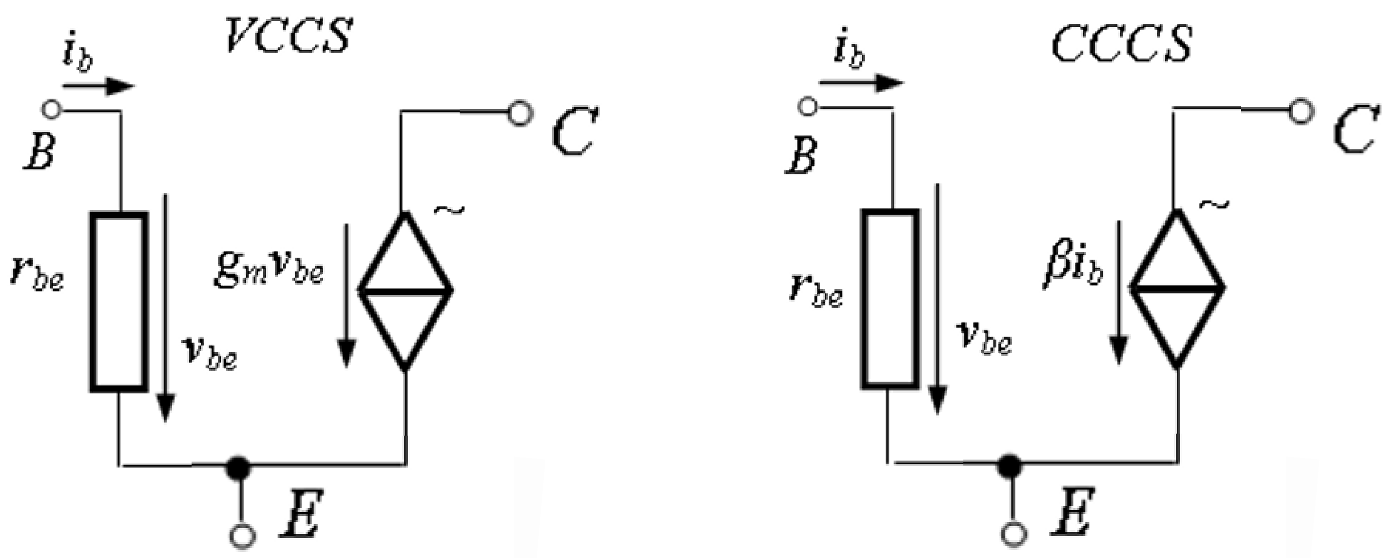
➢ Small-signal model & parameters - summary

$r_i = r_{be} = \frac{\beta}{g_m}$ $g_m = 40 I_C$ $r_o = r_{ce} = \frac{V_A}{I_C}$ $i_c = g_m v_{be} = 40 I_C v_{be}$

simplified *hybrid-π* models - $r_{ce} = \infty$

VCCS – voltage-controlled current source

CCCS – current-controlled current source



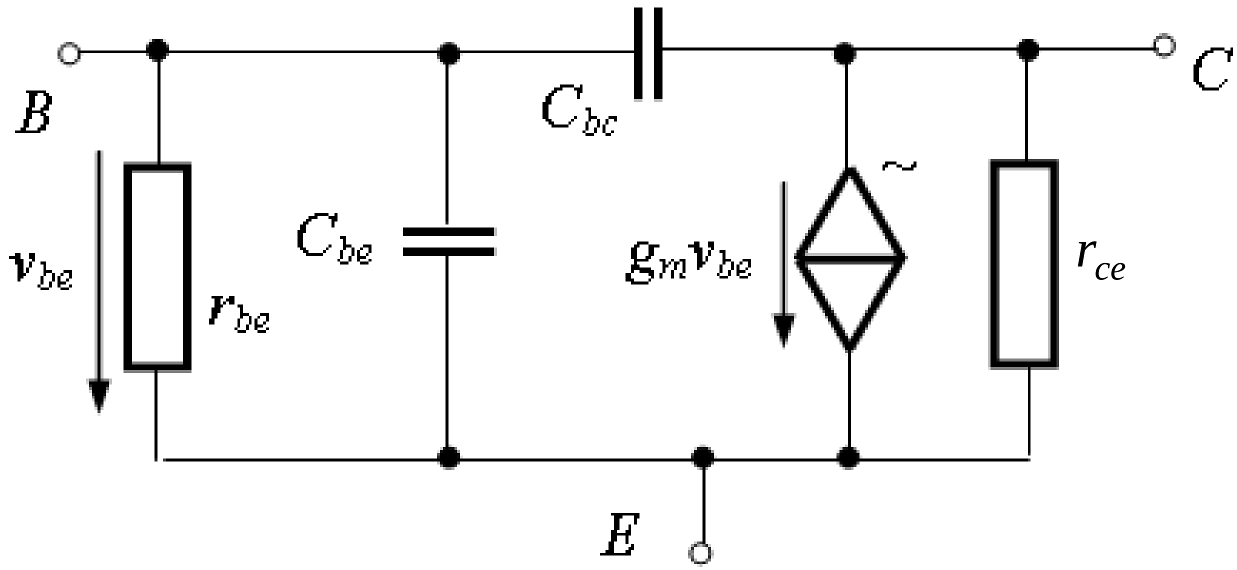
Valid for low & medium frequency

BJT small-signal model

➢ Small-signal model & parameters - summary

$$r_i = r_{be} = \frac{\beta}{g_m} \qquad g_m = 40I_C \qquad r_o = r_{ce} = \frac{V_A}{I_C} \qquad i_c = g_m v_{be} = 40I_C v_{be}$$

hybrid-π model - high frequency



Parasitic capacitances (pF or less) between terminals decrease the gain at high frequencies. How/why? We'll see next week!

➢ Small-signal model & parameters - examples

Assume low & medium frequency operation

Ex. 1 $\beta = 100,\ V_A = 100\ \text{V};\ I_C = 100\ \mu\text{A}$

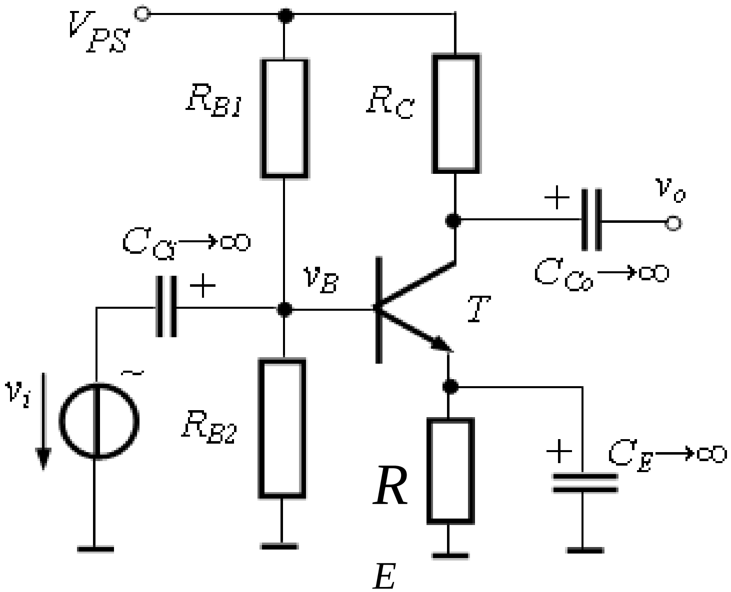
$g_m = 40 I_C = 4\ \text{mS}$ $r_{be} = \dfrac{\beta}{g_m} = 25\ \text{K}\Omega$ $r_{ce} = \dfrac{V_A}{I_C} = 1\ \text{M}\Omega$

Ex. 2 $\beta = 100,\ V_A = 100\ \text{V};\ I_C = 1\ \text{mA}$

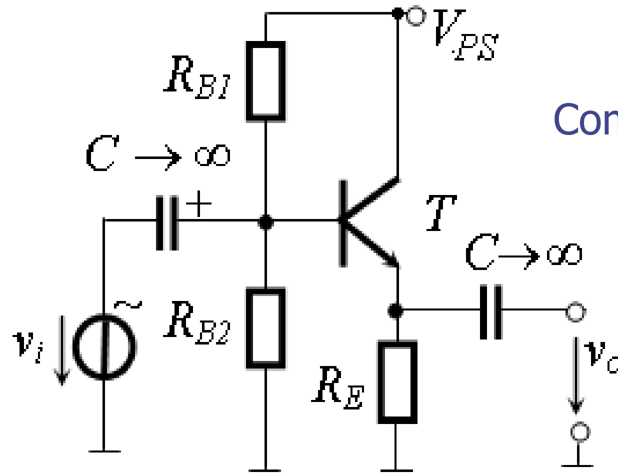
$g_m = 40 I_C = 40\ \text{mS}$ $r_{be} = \dfrac{\beta}{g_m} = 2.5\ \text{K}\Omega$ $r_{ce} = \dfrac{V_A}{I_C} = 100\ \text{K}\Omega$

BJT basic amplifiers

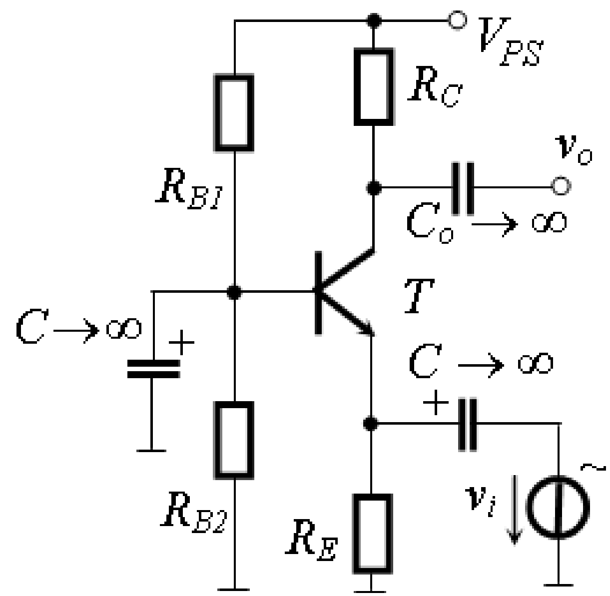
➢ Configurations



Common emitter (CE)



Common collector (CC)



Common base (CB)

How many possible configurations? What will they be called?

Why are the capacitors necessary?

How are the circuits analyzed?

BJT basic amplifiers

➢ Configurations

- Common emitter/collector/base

The terminal that is connected to ground in the small-signal equivalent schematic gives the name of the configuration.

- Why are the capacitors necessary?

Capacitors separate the variable signals from the dc ones (at the input, output, or in other points of the circuit).

The capacitances need to be high enough so that they can be considered short circuits at the operating frequency (their equivalent impedance << the series/parallel resistances connected with them).

On the dc equivalent schematic (the one used to determine the operating point *Q*), the capacitors are considered open circuits.

- How are the circuits analyzed?

See next slide(s)

➢ Transistor amplifier analysis - steps

Start with full amplifier circuit (transistor, resistors, capacitors, dc sources, ac sources)

1. Draw dc equivalent circuit
   - compute the operating point $Q\,(V_{CE};\ I_C)$
   - determine dc potentials in the three terminals of the transistor
   - compute small-signal parameters of the transistor: g_m r_{be} r_{ce}

C – open-circuit

2. Draw small-signal equivalent circuit
   - compute amplifier performance:

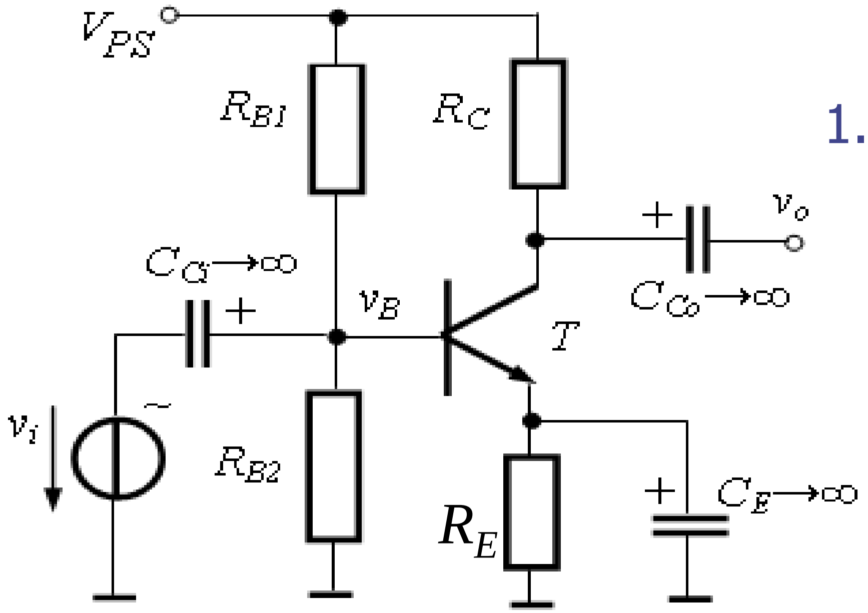
   gain A_v input resistance R_i output resistance R_o

C – short-circuit
dc sources - passive

3. Plot waveforms in various points of the amplifier
   - small-signal waveforms
   - full waveforms (dc + small-signal)

BJT basic amplifiers

➢ Common emitter (CE) basic amplifier



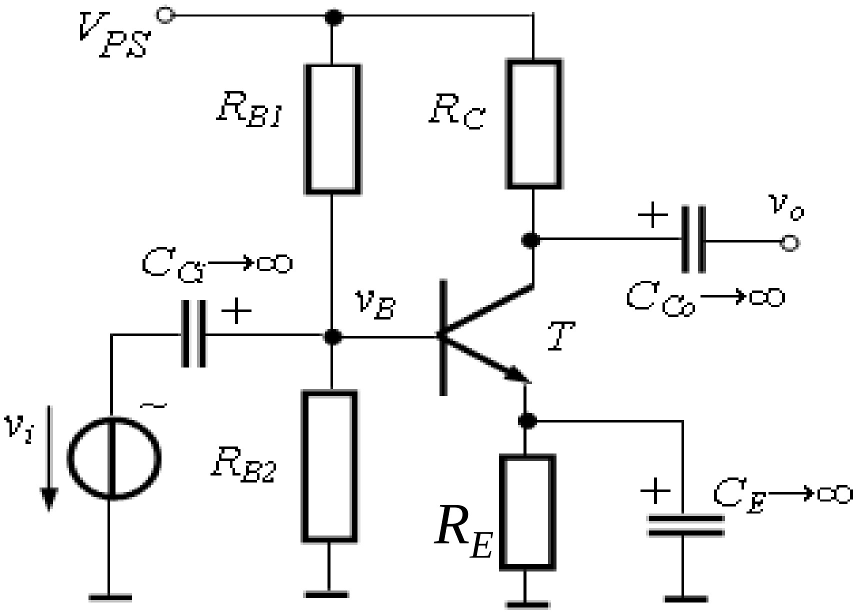
1. Draw dc equivalent circuit
   - compute the operating point $Q\,(V_{CE};\,I_C)$
   - determine dc potentials in the three terminals of the transistor
   - compute small-signal parameters of the transistor: g_m r_{be} r_{ce}

See Seminar 1 and C2

$$g_m = 40 I_C \qquad r_{be} = \frac{\beta}{g_m} \qquad r_{ce} = \frac{V_A}{I_C}$$

BJT basic amplifiers

➢ Common emitter (CE) basic amplifier

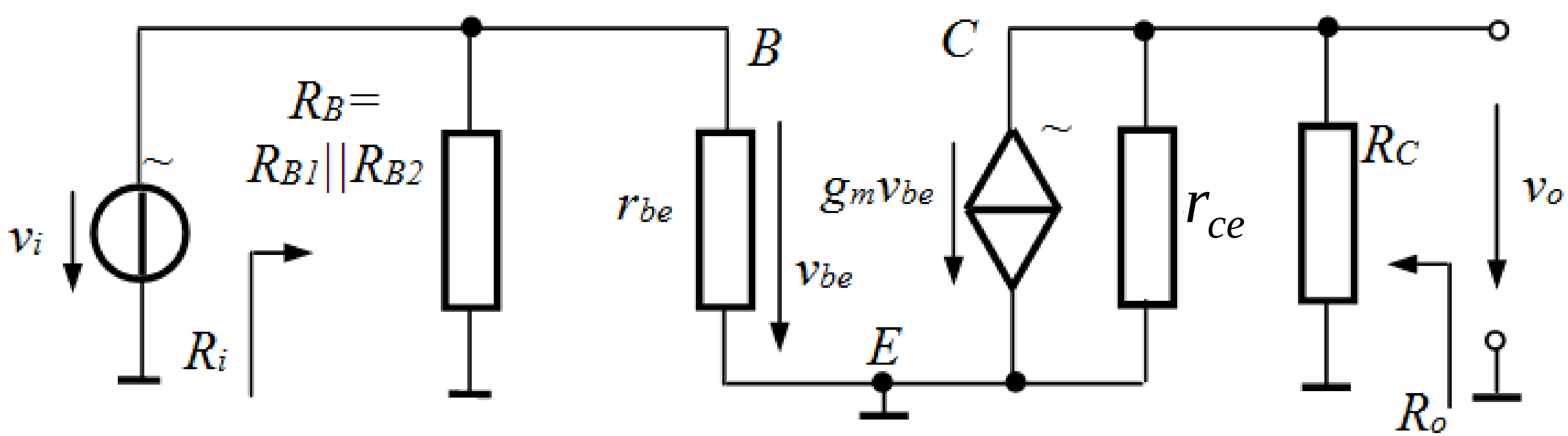


2. Draw small-signal equivalent circuit
   - compute amplifier performance:

     gain A_v

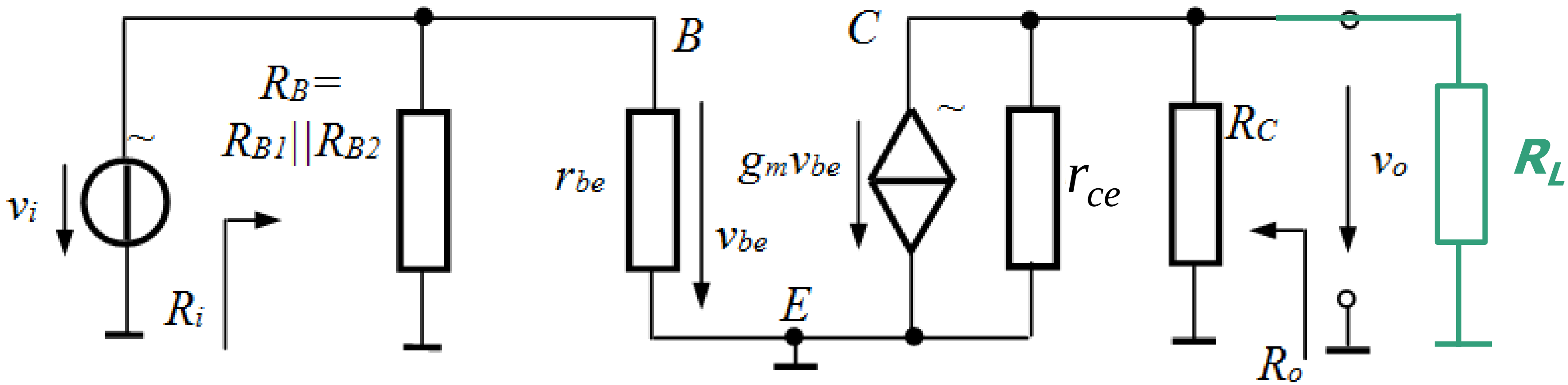
     input resistance R_i

     output resistance R_o



BJT basic amplifiers

➢ Common emitter (CE) basic amplifier



$$R_i = R_{B1}||R_{B2}||r_{be}$$

$$A_v = \frac{v_o}{v_i}$$

$$v_i = v_{be}$$

$$v_o = -g_m v_{be}(R_C||r_{ce})$$

$$A_v = -g_m(R_C||r_{ce})$$

Usually, $R_C \ll r_{ce}$, so $A_v \approx -g_m R_C$

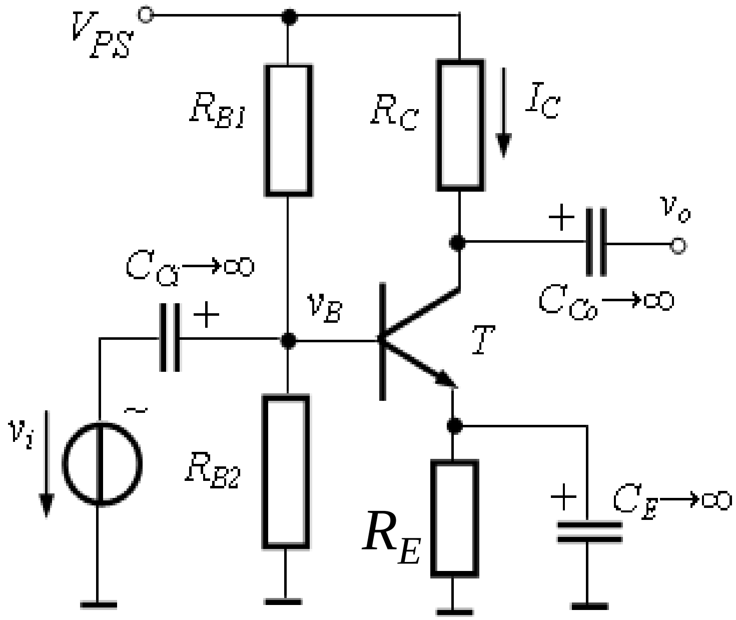
$$R_o = R_C||r_{ce} \approx R_C$$

$A_v < 0$ – inverting amplifier

If R_L is present:

$$A_v = -g_m(R_C||r_{ce}||\boldsymbol{R_L})$$
$$A_v \approx -g_m(R_C||\boldsymbol{R_L})$$

➢ Common emitter (CE) basic amplifier - example

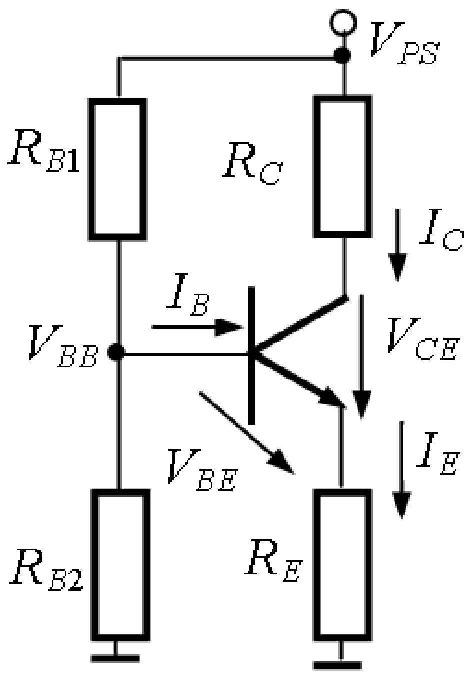


V_{PS} = 12 V
R_{B1} = 49 KΩ R_{B2} = 22 KΩ
R_C = 2 KΩ, R_E = 2.5 KΩ
β = 100 V_A = 100 V
$V_{BE,\ on}$ = 0.7 V

a) Draw the dc equivalent circuit. Determine $Q(V_{CE}, I_C)$.
b) Compute the small-signal parameters of the transistor, g_m, r_{be}, r_{ce}.
c) Draw the small-signal equivalent circuit. What is the configuration of the stage? Justify.
d) Compute the gain, input and output resistances.
e) For $v_i(t)$ - triangular wave, 10 mV amplitude, plot $v_B(t)$, $v_o(t)$, $v_C(t)$, $v_E(t)$.
f) What changes when R_L = 10 KΩ is connected at the output of the circuit? Recompute any of the previously determined values that are subject to change.

➢ Common emitter (CE) basic amplifier - example

a) Draw the dc equivalent circuit. Determine $Q(V_{CE}, I_C)$.
b) Compute the small-signal parameters of the transistor, g_m, r_{be}, r_{ce}.



$$Q(6.6\text{ V}; 1.2\text{ mA})$$

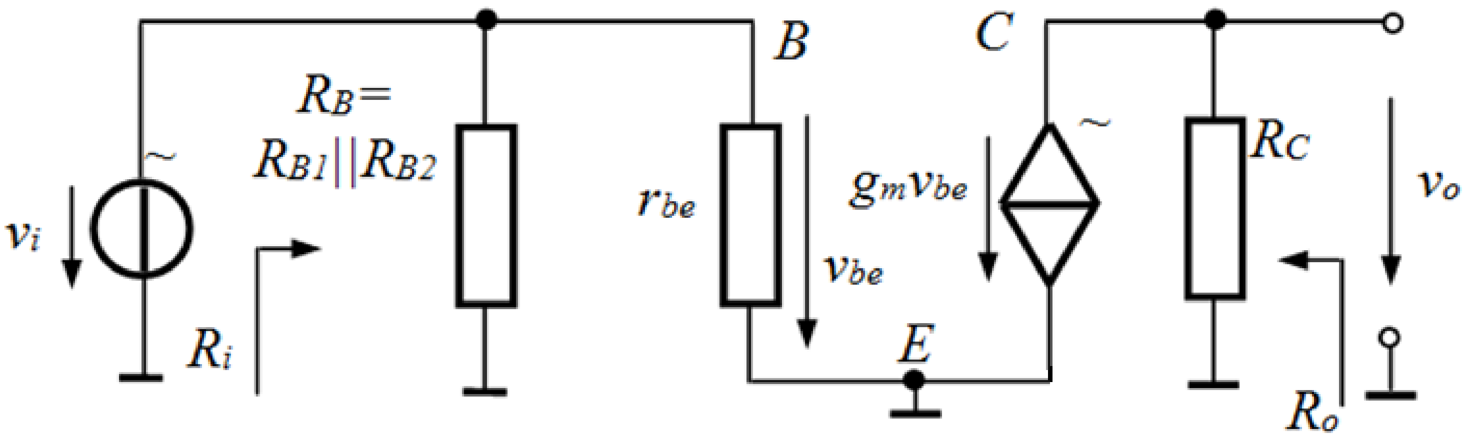
$$g_m = 40 I_c = 48\text{ mS}$$

$$r_{be} = \frac{\beta}{g_m} = 2.08\text{ K}\Omega$$

$$r_{ce} = \frac{V_A}{I_C} = 83.33\text{ K}\Omega$$

➢ Common emitter (CE) basic amplifier - example

d) Draw the small-signal equivalent circuit. What is the configuration of the stage? Justify.
e) Compute the gain, input and output resistances.



$R_C \ll r_{ce}$

$$A_v \approx -g_m R_C = -48 \cdot 2 = -96$$

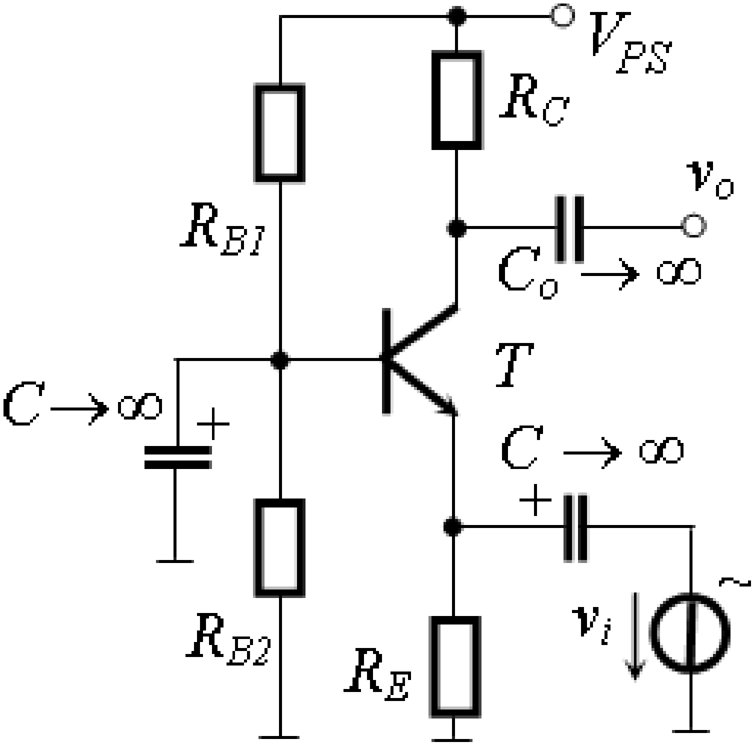
$$R_i = R_B = R_{B1}||R_{B2}||r_{be} = 1.83\ \text{K}\Omega$$

$$R_o = R_c||r_{ce} \approx R_c = 2\ \text{K}\Omega$$

common emitter, terminal E is connected to ground

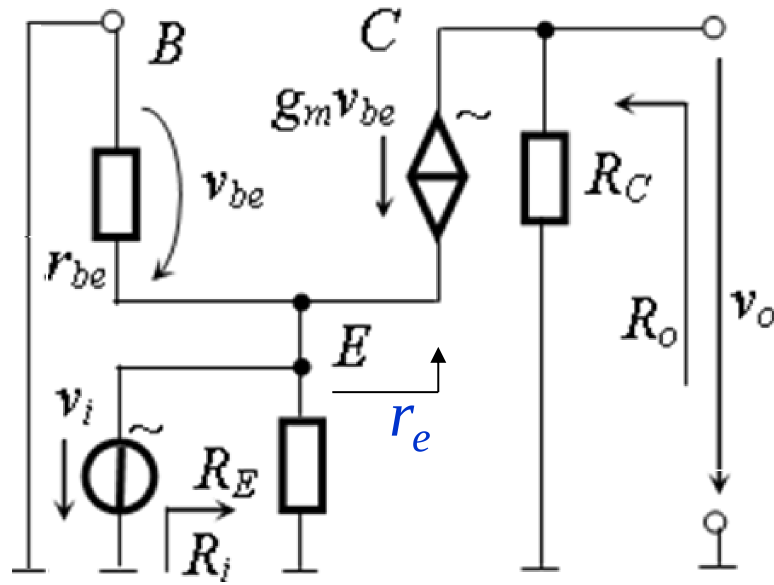
BJT basic amplifiers

➢ Common base (CB) basic amplifier



$R_o = R_C$

$A_v = g_m R_C$



T for small-signal – simplified hybrid-π model

r_e - resistance seen into E when $v_i = 0$

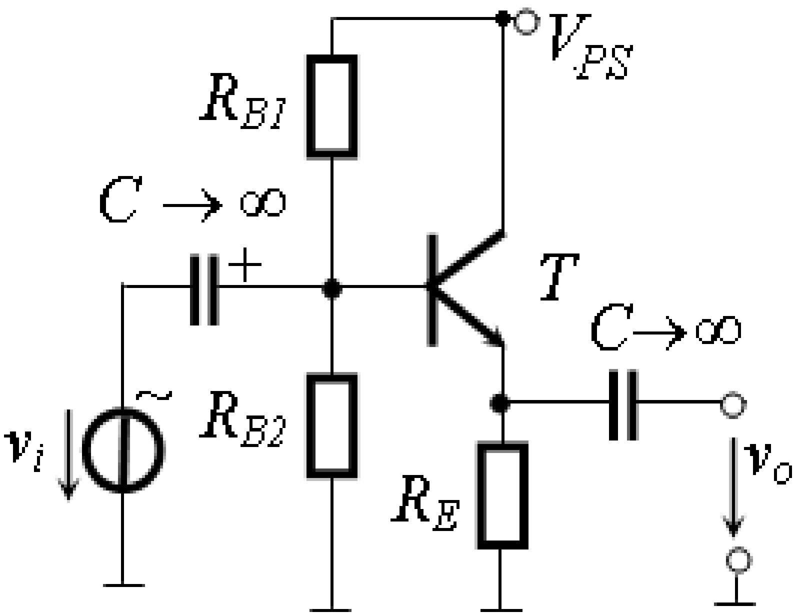
$$r_e = \frac{-v_{be}}{-v_{be}/r_{be} - g_m v_{be}} = \frac{1}{\frac{1}{r_{be}} + g_m} = \frac{1}{\frac{g_m}{\beta} + g_m} \approx \frac{1}{g_m}$$

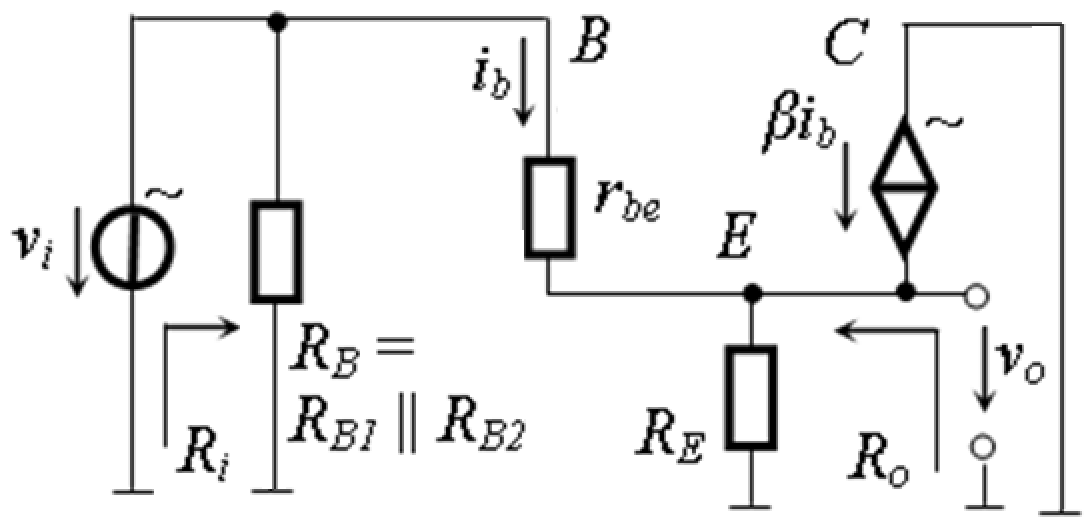
$$v_o = -R_C g_m v_{be}$$
$$v_i = -v_{be}$$

$$A_v = \frac{-R_C g_m v_{be}}{-v_{be}} = g_m R_C$$

$$R_i = R_E || r_e = R_E || \frac{1}{g_m} \approx \frac{1}{g_m}$$

➢ Common collector (CC) basic amplifier





T for small-signal – simplified hybrid-π model

$$A_v = \frac{v_o}{v_i} = \frac{g_m R_E}{1 + g_m R_E} \approx 1$$

$$A_v \approx 1$$

$$R_i = R_B \| (r_{be} + (\beta + 1) R_E)$$

$$R_o = R_E \| \frac{1}{g_m} \approx \frac{1}{g_m}$$

Voltage follower (emitter follower)

➢ BJT basic amplifiers - summary

Common emitter CE	Common base CB	Common collector CC
$A_v = -g_m(R_C \| \| r_{ce}) \approx -g_m R_C$	$A_v \approx g_m R_C$	$A_v \approx 1$
$R_i = R_{B1} \| \| R_{B2} \| \| r_{be}$	$R_i \approx \frac{1}{g_m}$	$R_i = R_i = R_B \| \| (r_{be} + (\beta + 1) R_E)$
$R_o = R_C$	$R_o = R_C$	$R_o \approx \frac{1}{g_m}$

Which one is best? What does best mean for gain, input/output resistance?

Transistor amplifiers - requirements

- high gain
- high input resistance
- low output resistance

Common emitter CE

$$A_v = -g_m(R_C||r_{ce}) \approx -g_m R_C$$

high gain (absolute value)

$$R_i = R_{B1}||R_{B2}||r_{be}$$

rather low input resistance

Common collector CC

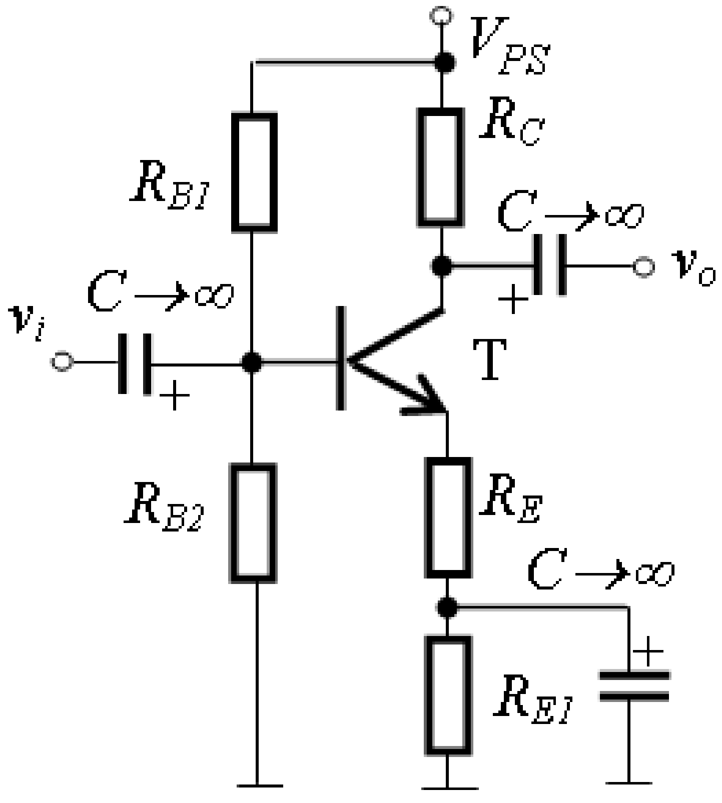
$$A_v \approx 1$$

low (unitary) gain

$$R_i = R_i = R_B || (r_{be} + (\beta + 1) R_E)$$

high input resistance

➢ Modified CE (emitter degeneration) amplifier



$$A_v \approx -\frac{g_m R_C}{1 + g_m R_E}$$

If $g_m R_E >> 1$ $\quad A_v \approx -\frac{R_C}{R_E}$

➢ stable gain, independent of transistor parameters

$$R_i = R_B || [r_{be} + (\beta + 1) R_E]$$

$$R_o = R_C$$

$R_E + R_{E1}$ appear in dc – sets I_C

R_E appears in ac – sets the gain

➢ Summary of single-stage transistor amplifiers

Config	Gain	Input resistance	Output resistance
CS	$-g_m(R_D\|\|r_{ds}) \approx -g_m R_D$	R_G	$R_D\|\|r_{ds} \approx R_D$
CE	$-g_m(R_C\|\|r_{ce}) \approx -g_m R_C$	$R_{B1}\|\|R_{B2}\|\|r_{be}$	R_C
CG	$g_m(R_D\|\|r_{ds}) \approx g_m R_D$	$\approx 1/g_m$	$R_D\|\|r_{ds} \approx R_D$
CB	$g_m(R_C\|\|r_{ce}) \approx g_m R_C$	$\approx 1/g_m$	R_C
CD	≈ 1	R_G	$\approx R_S\|\|1/g_m$
CC	≈ 1	$R_B\|\|\left[r_{be} + (\beta + 1)R_E\right]$	$\approx 1/g_m$
Emitter degeneration	$\approx -R_C/R_E$	$R_B\|\|\left[r_{be} + (\beta + 1)R_E\right]$	R_C

Summary

- BJT small-signal model
- BJT basic amplifiers
- Comparison and analysis

Next week: Frequency response. Current sources and current mirrors.