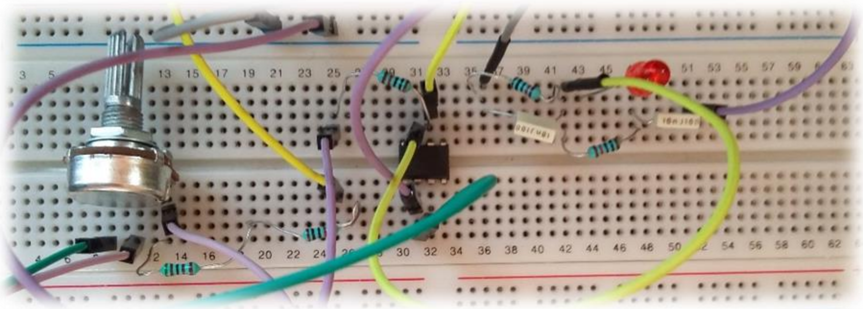


FUNDAMENTAL ELECTRONIC CIRCUITS

Assist. prof. Laura-Nicoleta IVANCIU, Ph.D.

C5 – Frequency response. Current sources and current mirrors.

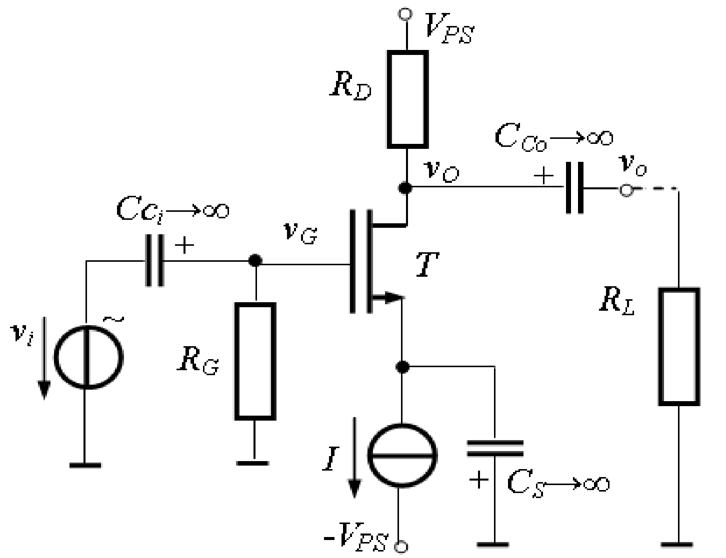
Contents

- Frequency response of transistor amplifiers
  - qualitative analysis for CS amplifier
  - quantitative analysis for CS and CE amplifiers
  - cascode amplifiers
- Current sources
- Current mirrors

➢ CS amplifier - revisited

Mid-frequency complete circuit

Coupling capacitors: C_{Ci} C_{Co} C_S



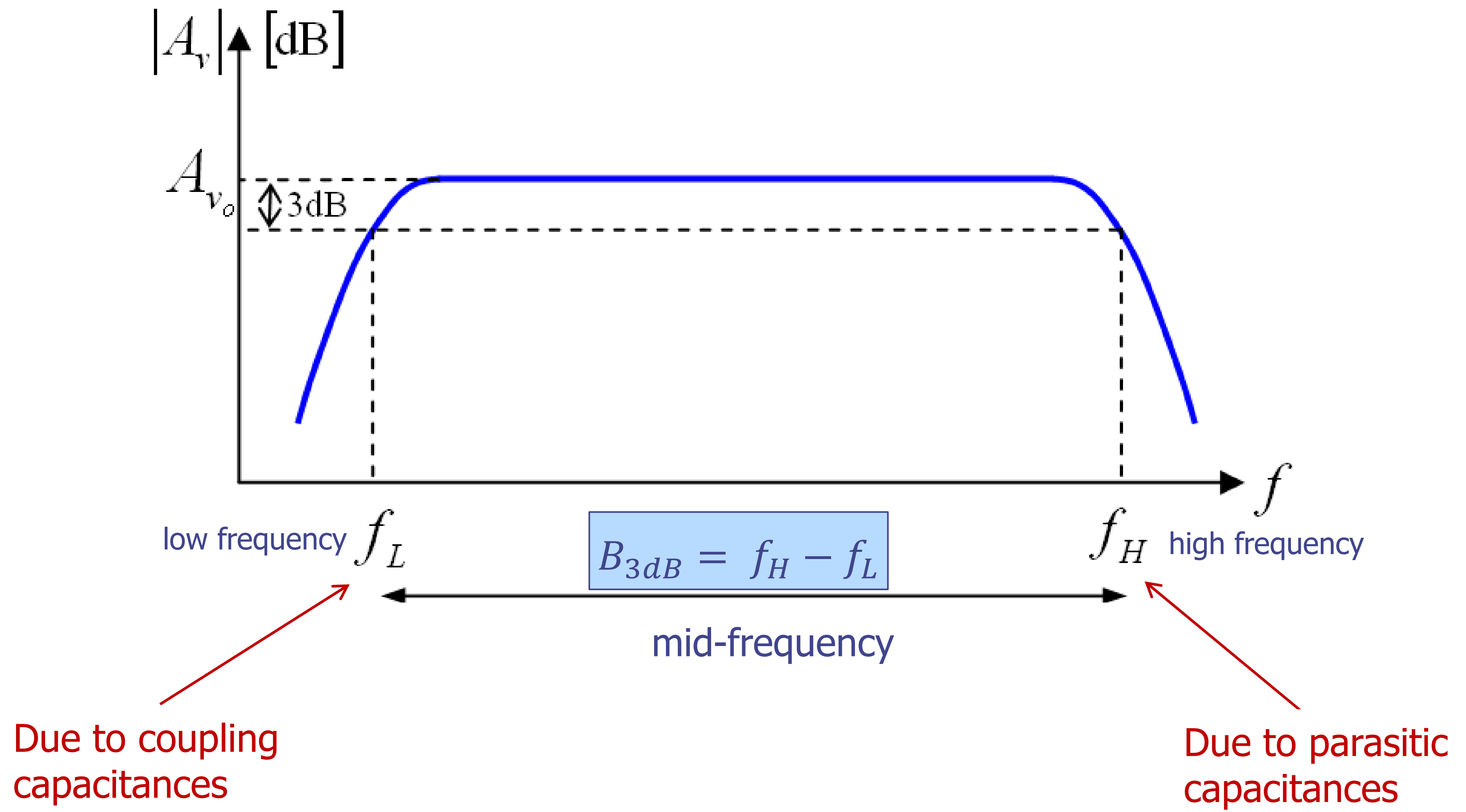
Frequency response of transistor amplifiers

➢ Premises

- mid-frequency:
  - coupling capacitors → short-circuits
  - internal parasitic capacitances → open-circuits
- low-frequency:
  - coupling capacitors → equivalent impedances
  - internal parasitic capacitances → open-circuits
- high-frequency:
  - coupling capacitors → short-circuits
  - internal parasitic capacitances → equivalent impedances

The effect of the output resistance of the input signal source and load resistance must be taken into account.

➢ Gain – frequency plot of CS amplifier



➢ Gain – frequency plot of CS amplifier

- mid-frequency:

$$|A_v(j\omega)| = const$$

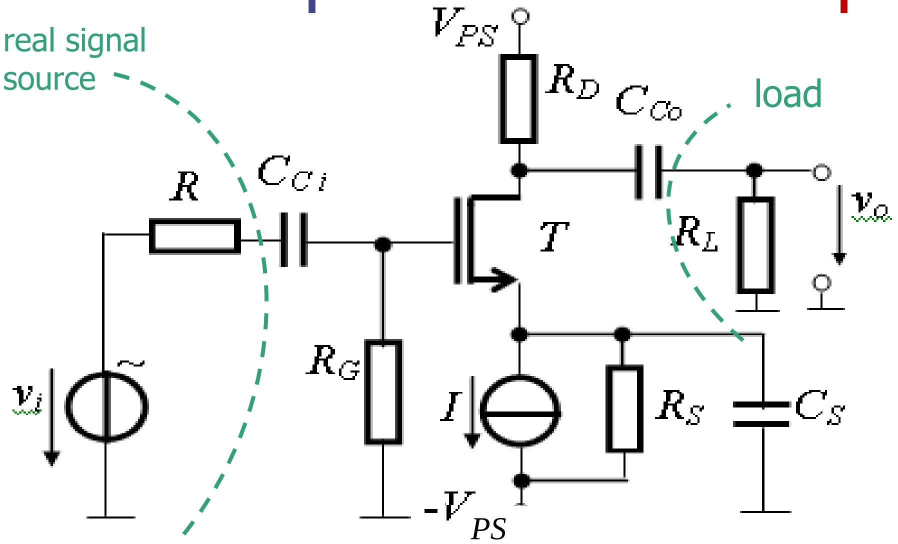
- low-frequency:
  - coupling capacitors determine f_L

$$f \downarrow, |A_v(j\omega)| \downarrow$$

- high-frequency:
  - internal parasitic capacitances determine f_H

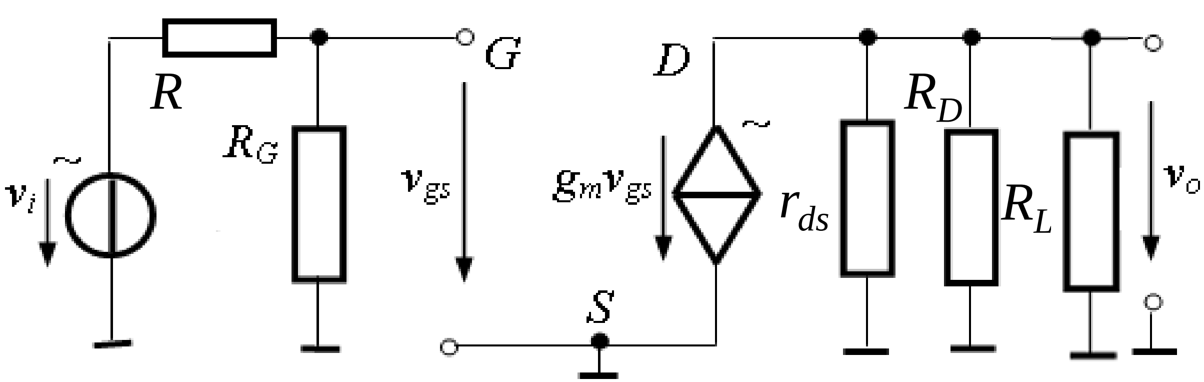
$$f \uparrow, |A_v(j\omega)| \downarrow$$

➢ CS amplifier – mid-frequency



- **No capacitors** in the small-signal equivalent circuit
- **Frequency independent** behavior

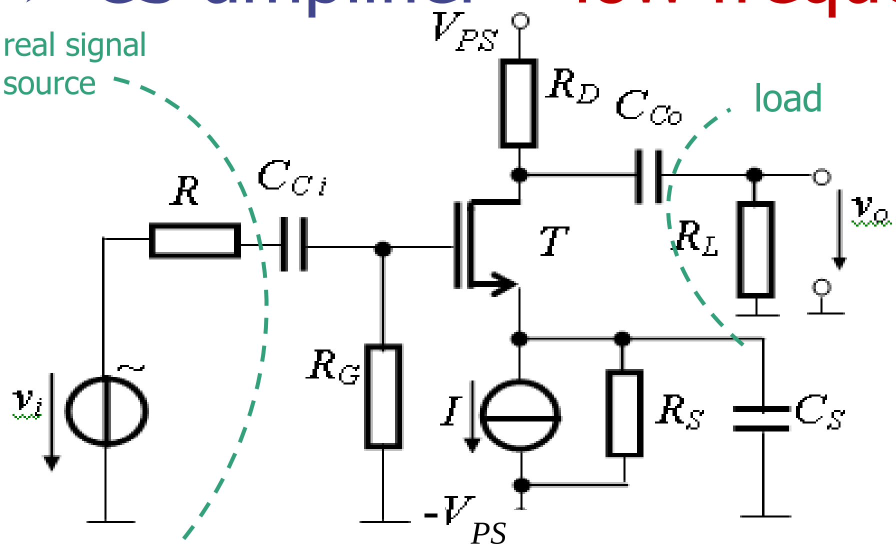
$$|A_v(j\omega)| = const$$



Frequency response of transistor amplifiers

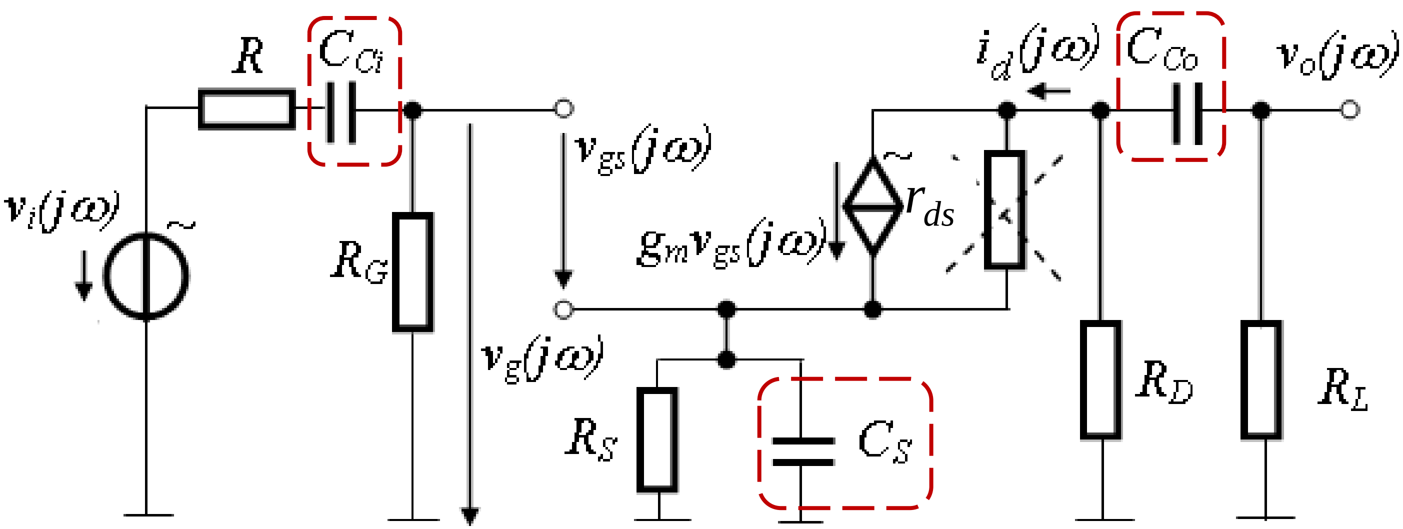
OPTIONAL

➢ CS amplifier – low frequency



- **Coupling capacitors** in the small-signal equivalent circuit
- **Frequency dependent** behavior (high-pass type)

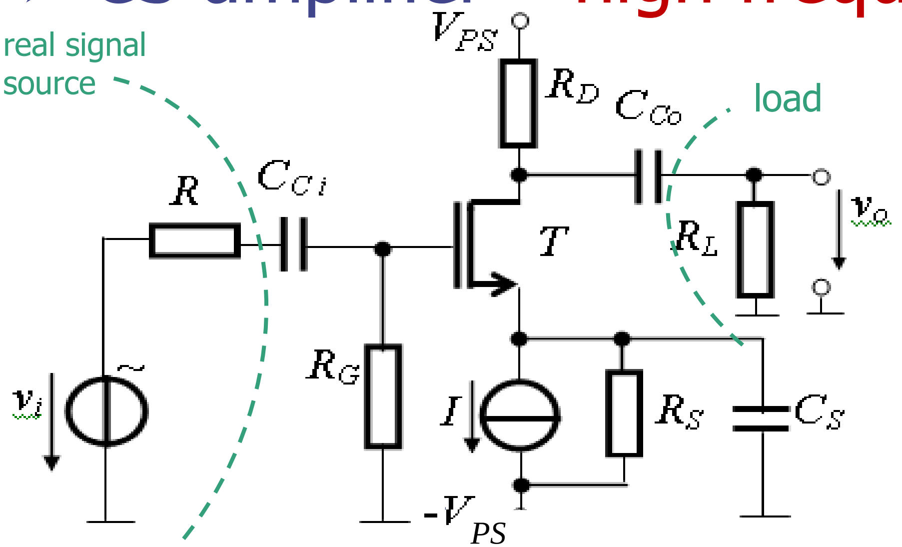
$$f \downarrow, |A_v(j\omega)| \downarrow$$



Sets f_L – low cutoff frequency

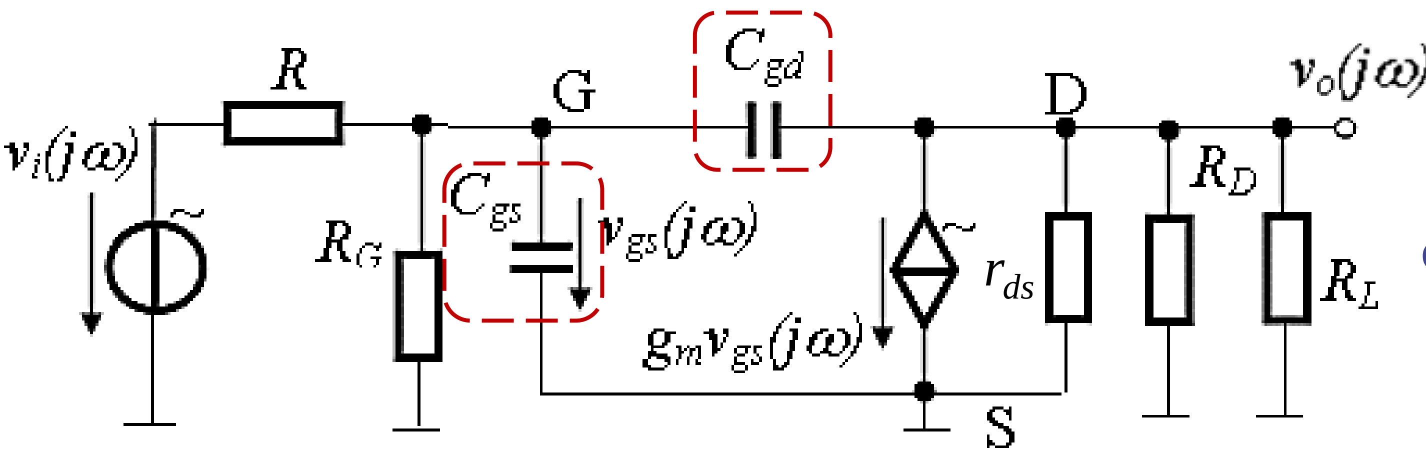
OPTIONAL

➢ CS amplifier – high frequency



- **Parasitic capacitors** in the small-signal equivalent circuit
- **Frequency dependent** behavior (low-pass type)

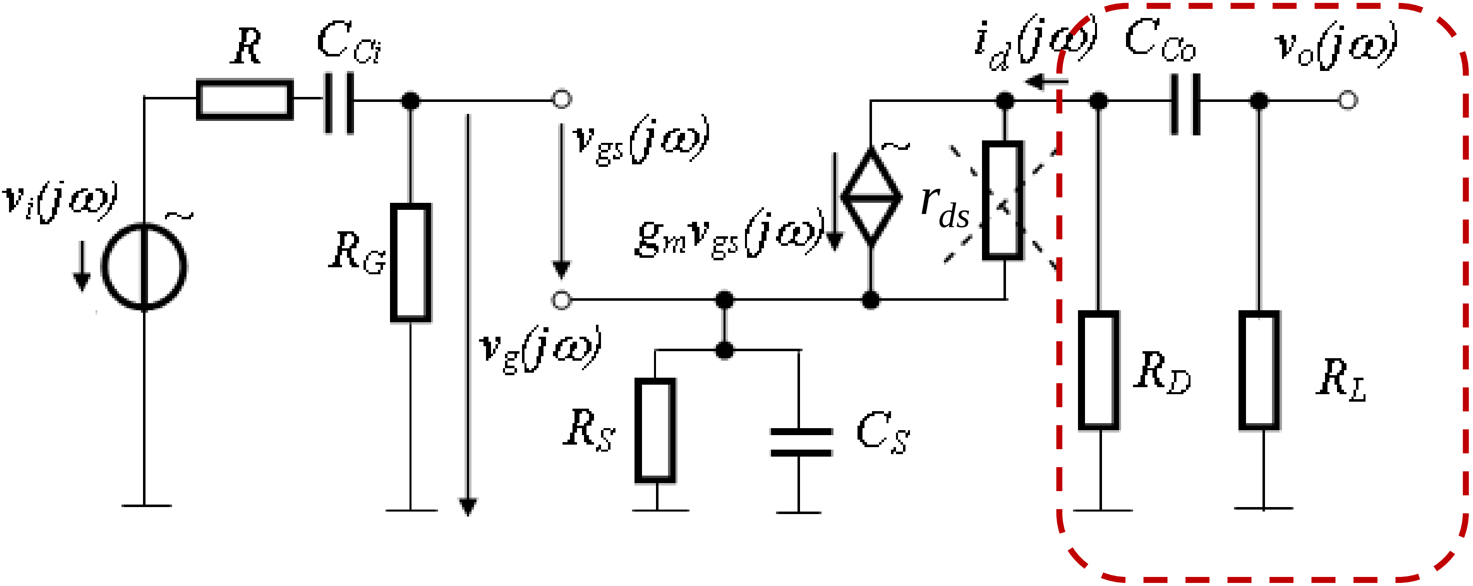
$f \uparrow, |A_v(j\omega)| \downarrow$



Sets f_H – high cutoff frequency

OPTIONAL

➢ CS amplifier – low frequency analysis



$$A_v(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}$$

$$v_g(j\omega) = F_i(j\omega) \cdot v_i(j\omega)$$

$$i_d(j\omega) = F_S(j\omega) \cdot v_g(j\omega)$$

$$v_o(j\omega) = F_o(j\omega) \cdot i_d(j\omega)$$

$$A_v(j\omega) = F_o(j\omega) \cdot F_S(j\omega) \cdot F_i(j\omega)$$

OPTIONAL

CS amplifier – low frequency analysis

$$A_v(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)} = F_o(j\omega) \cdot F_S(j\omega) \cdot F_i(j\omega)$$

- each coupling capacitor determines a low cutoff frequency

$$F_i(j\omega) = \frac{v_g(j\omega)}{v_i(j\omega)} = \frac{j\omega R_G C_{C_i}}{1 + j\omega (R + R_G) C_{C_i}}$$

$$f_{Li} = \frac{1}{2\pi (R + R_G) C_{Ci}}$$

$$F_S(j\omega) = \frac{i_d(j\omega)}{v_g(j\omega)} = \frac{g_m v_{gs}(j\omega)}{v_{gs}(j\omega) + g_m v_{gs}(j\omega)\left(R_S || \frac{1}{j\omega C_S}\right)}$$

$$f_{LS} = \frac{1}{2\pi \left(\frac{1}{g_m} || R_S\right) C_S}$$

$$F_o(j\omega) = \frac{v_o(j\omega)}{i_d(j\omega)}$$

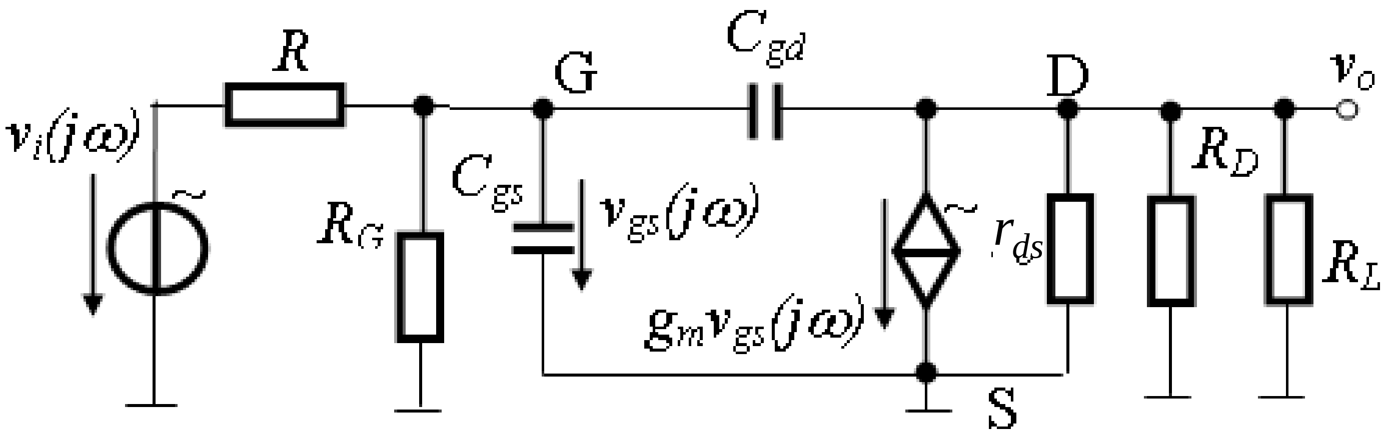
$$f_{Lo} = \frac{1}{2\pi (R_D + R_L) C_{Co}}$$

Dominant pole: max(f_{Li}, f_{Ls}, f_{Lo}), if the nearest pole or zero is at least a decade away.

Usually given by ***f_{Ls}***, for equal coupling capacitances

OPTIONAL

CS amplifier – high frequency analysis

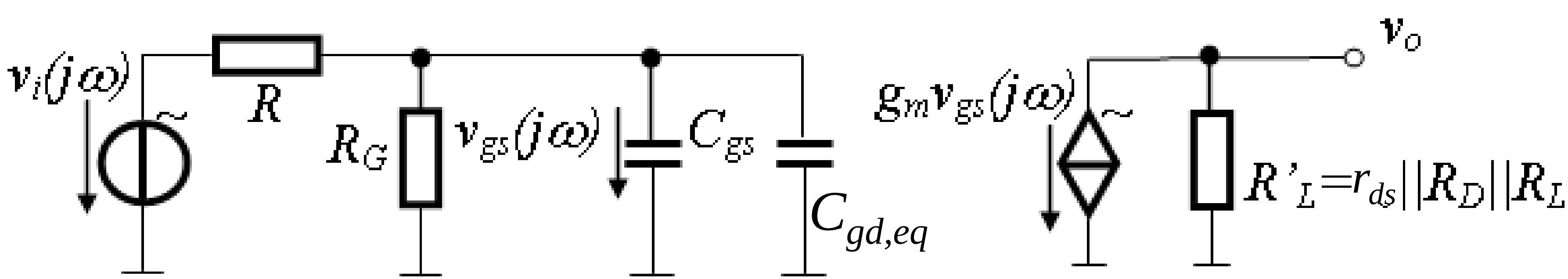


$$A_v(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}$$

C_{ds} - not shown, since it generates a pole at a much higher frequency that the one generated by C_{gs} and C_{gd}

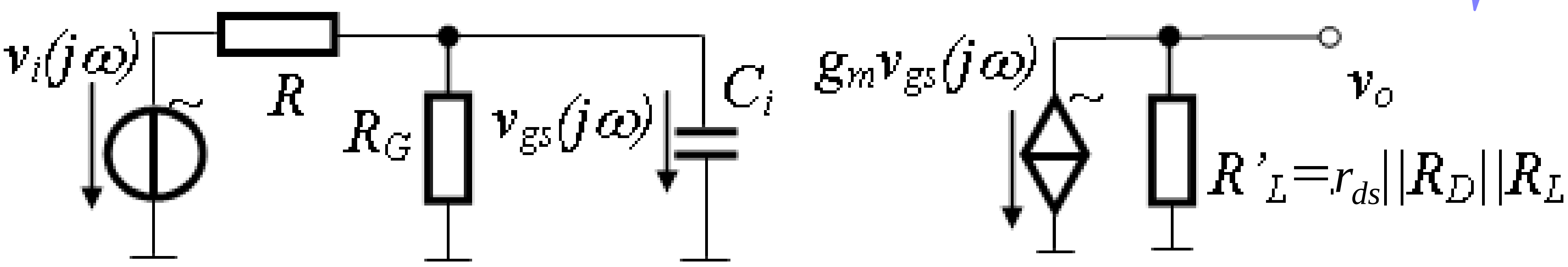
$$C_{gd,_{eq}} = (1 - a_v)C_{gd} \qquad a_v = -g_m R_L' = -g_m(R_L||R_D||r_{ds}) \qquad C_{gd,_{eq}} = (1 + g_m R'_L)C_{gd}$$

C_{gd} is reflected at the input according to Miller's theorem



OPTIONAL

➢ CS amplifier – high frequency analysis



$$C_i = C_{gs} + C_{gd,_{\mathrm{eq}}} = C_{gs} + (1 + g_m R'_L)C_{gd}$$

$$A_v(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)} = -\frac{R_G}{R + R_G} g_m R'_L \frac{1}{1 + j\omega(R||R_G)C_i}$$

$$A_{vo} = -\frac{R_G}{R + R_G} g_m R'_L$$

$$f_H = \frac{1}{2\pi(R||R_G)C_i}$$

CS amplifier – frequency analysis summary

OPTIONAL

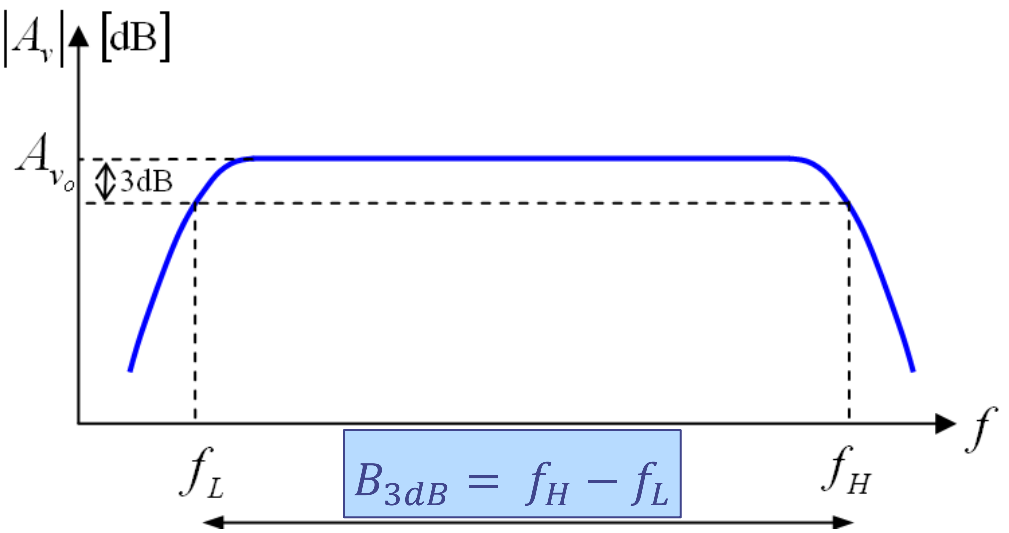
$$f_{Li} = \frac{1}{2\pi(R + R_G)C_{Ci}}$$

$$f_H = \frac{1}{2\pi(R||R_G)C_i}$$

$$f_{Ls} = \frac{1}{2\pi\left(\frac{1}{g_m}||R_S\right)C_S}$$

$$f_{Lo} = \frac{1}{2\pi(R_D + R_L)C_{Co}}$$

$$f_L = \max(f_{Li}, f_{Ls}, f_{Lo})$$



$$B_{3dB} = f_H - f_L$$

CS amplifier – frequency analysis example

OPTIONAL

$C_{ci} = C_{Co} = C_s = 10\ \mu\text{F},\ R = 20\ \text{K}\Omega,\ R_G = 2\ \text{M}\Omega,\ R_D = 10\ \text{K}\Omega,\ R_L = 20\ \text{K}\Omega,\ R_s = 10\ \text{K}\Omega,\ I = 400\ \mu\text{A}$

$K = 100\mu\text{A/V}^2,\ (W/L) = 18,\ V_A = 100\ \text{V}$

$C_{gs} = C_{gd} = 1\ \text{pF} \ @\ I = 400\ \mu\text{A}$

Compute f_L, f_H, A_{vo}. Draw the gain-frequency plot.

$$g_m = 1.2\ \text{mS} \qquad r_o = 250\ \text{K}\Omega$$

$$f_{Li} = \frac{1}{2\pi(R+R_G)C_{Ci}} = \frac{1}{2\pi(20+2000)10^3 \cdot 10 \cdot 10^{-6}} \cong 8\ \text{mHz}$$

$$f_{Ls} = \frac{1}{2\pi\left(\frac{1}{g_m}||R_S\right)C_S} = \frac{1}{2\pi\left(\frac{1}{1.2}||10\right)10^3 \cdot 10 \cdot 10^{-6}} \cong 21\ \text{Hz}$$

$$f_{Lo} = \frac{1}{2\pi(R_D+R_D)} = \frac{1}{2\pi(10+20)10^3 \cdot 10 \cdot 10^{-6}} \cong 0.5\ \text{Hz}$$

$f_L = 21\ \text{Hz}$

CS amplifier – frequency analysis example

OPTIONAL

$C_{ci}=C_{Co}=C_s=10\ \mu\text{F},\ R=20\ \text{K}\Omega,\ R_G=2\ \text{M}\Omega,\ R_D=10\ \text{K}\Omega,\ R_L=20\ \text{K}\Omega,\ R_s=10\ \text{K}\Omega,\ I=400\ \mu\text{A}$

$K=100\mu\text{A/V}^2,\ (W/L)=18,\ V_A=100\ \text{V}$

$C_{gs}=C_{gd}=1\ \text{pF @ } I=400\ \mu\text{A}$

Compute f_L, f_H, A_{vo}. Draw the gain-frequency plot.

$$A_{vo}=-\frac{R_G}{R+R_G}g_mR_L'=-\frac{2}{0.02+2}\cdot 1.2\cdot 6.5=-7.7$$

$$|A_{vo}|_{\text{dB}}=20\log(7.7)=17.7$$

$$C_i=C_{gs}+[1+g_m(r_o||R_D||R_L)]C_{gd}=1+[1+1.2(250||10||20)]\cdot 1\cong 9.8\ \text{pF}$$

$$f_H=\frac{1}{2\pi(R||R_G)C_i}=\frac{1}{2\pi(20||2000)\cdot 10^3\cdot 9{,}8\cdot 10^{-12}}=820\ \text{KHz}$$

CS amplifier – frequency analysis example

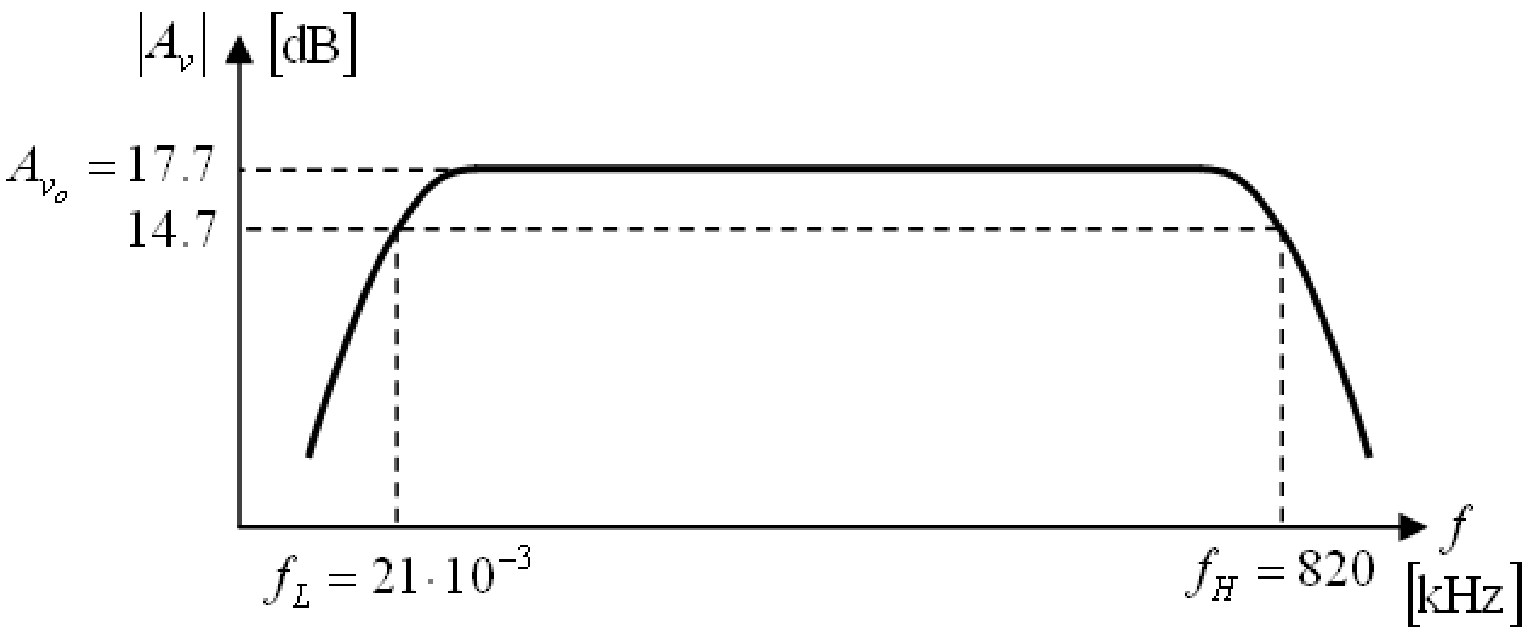
OPTIONAL

$C_{ci} = C_{Co} = C_s = 10\ \mu F$, $R = 20\ K\Omega$, $R_G = 2\ M\Omega$, $R_D = 10\ K\Omega$, $R_L = 20\ K\Omega$, $R_s = 10\ K\Omega$, $I = 400\ \mu A$

$K = 100 \mu A/V^2$, $(W/L) = 18$, $V_A = 100\ V$

$C_{gs} = C_{gd} = 1\ pF$ @ $I = 400\ \mu A$

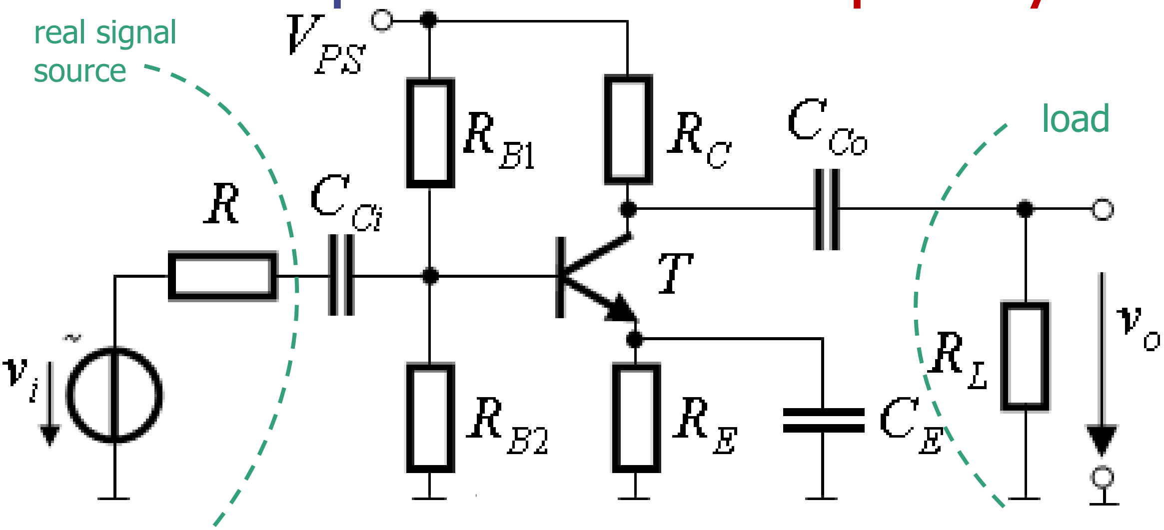
Compute f_L, f_H, A_{vo}. Draw the gain-frequency plot.



Frequency response of transistor amplifiers

OPTIONAL

➢ CE amplifier – low frequency



$$f_{Li} = \frac{1}{2\pi(R + R_i)C_{Ci}} = \frac{1}{2\pi(R + R_B||r_{be})C_{Ci}}$$

$$R_B = R_{B1}||R_{B2}$$

$$f_{Le} = \frac{1}{2\pi(R'||R_E)C_E} = \frac{1}{2\pi R'_E C_E}$$

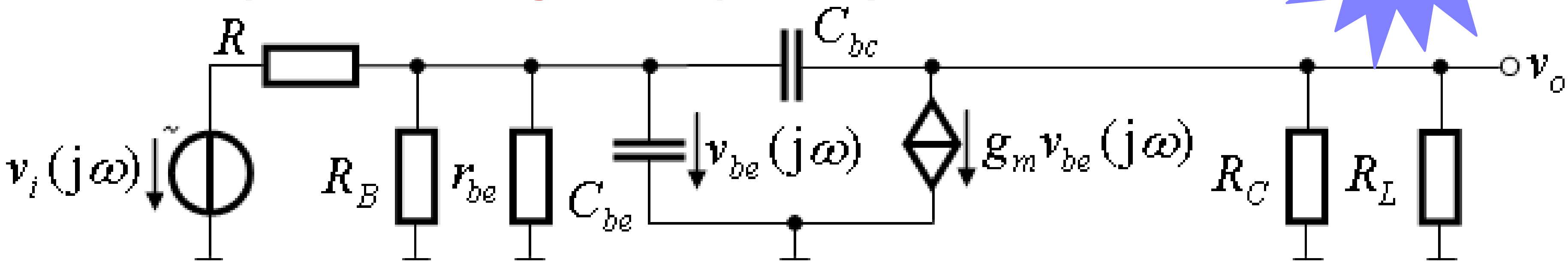
$$R'_E = R_E||R' = R_E||\frac{r_{be} + R_B||R}{\beta + 1}$$

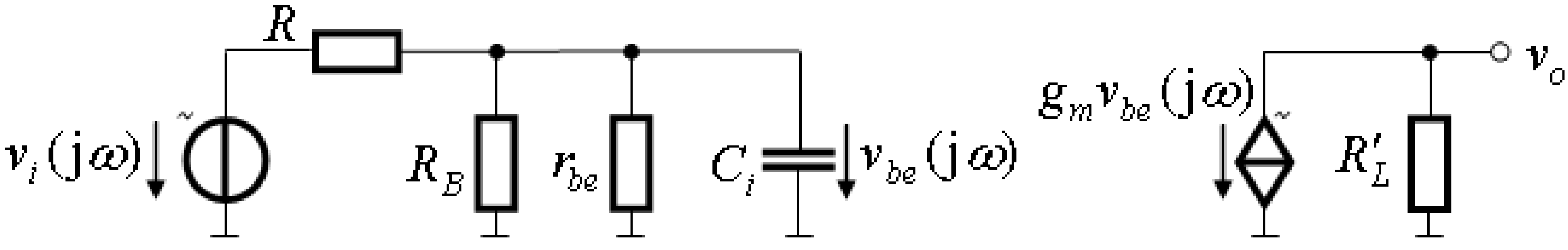
$$f_{Lo} = \frac{1}{2\pi(R_O + R_L)C_{Co}} = \frac{1}{2\pi(R_C + R_L)C_{Co}}$$

$$f_L = \max(f_{Li}, f_{Le}, f_{Lo})$$

OPTIONAL

➢ CE amplifier – high frequency





$$A_v(\mathrm{j}\omega) = -\frac{R_B||r_{be}}{R + R_B||r_{be}} g_m(R_C||R_L) \frac{1}{1 + \mathrm{j}\omega R'_i C_i}$$

$$A_{vo} = -\frac{R_B||r_{be}}{R + R_B||r_{be}} g_m(R_C||R_L)$$

$$R'_i = r_{be}||R_B||R$$

$$C_i = C_{be} + (1 + g_m R'_L)C_{bc}$$

$$f_H = \frac{1}{2\pi(r_{be}||R_B||R)C_i}$$

OPTIONAL

➢ CS & CE amplifiers - summary

CS

$$A_{vo} = -\frac{R_G}{R + R_G} g_m R'_L$$

$$f_H = \frac{1}{2\pi(R||R_G)C_i}$$

$$C_i = C_{gs} + C_{gd,_{eq}} = C_{gs} + (1 + g_m R'_L)C_{gd}$$

CE

$$A_{vo} = -\frac{R_B||r_{be}}{R + R_B||r_{be}} g_m (R_C||R_L)$$

$$f_H = \frac{1}{2\pi(r_{be}||R_B||R)C_i}$$

$$C_i = C_{be} + (1 + g_m R'_L)C_{bc}$$

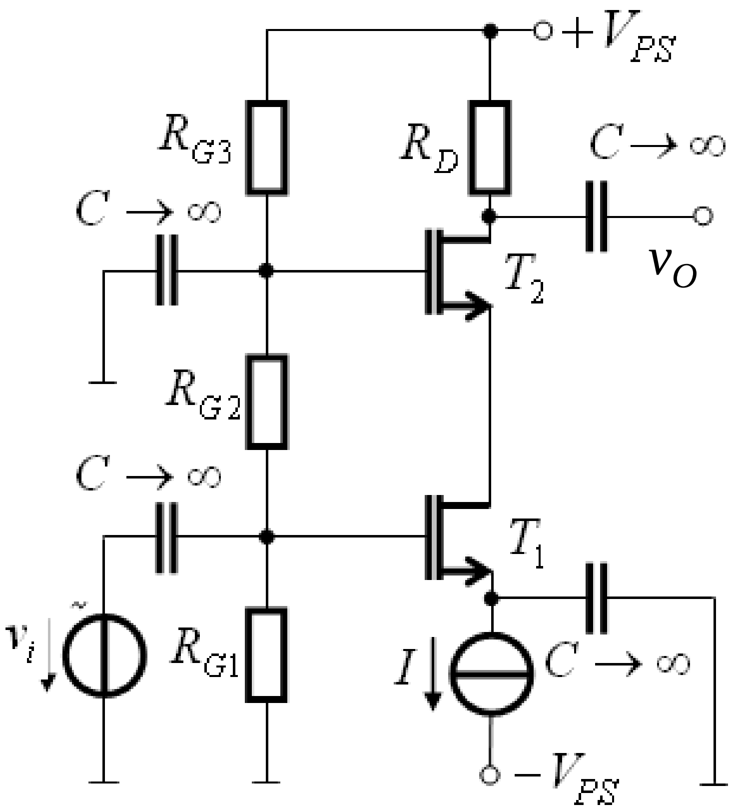
Problem: when the gain increases, the parasitic capacitance reflected at the input also increases, so the high cutoff frequency decreases

$$A_{vo} \uparrow, C_i \uparrow, f_H \downarrow$$

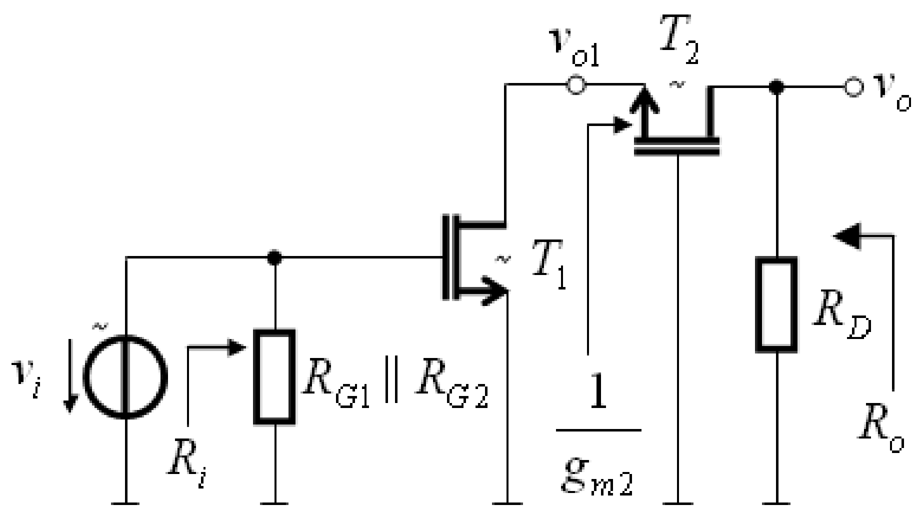
Solution?

Cascode amplifiers: CS+CG or CE+CB, to obtain a wide bandwidth

➢ Cascode amplifiers – CS + CG – medium frequency



same current through T_1, T_2



$$R_i = R_{G1}||R_{G2} = R_G$$

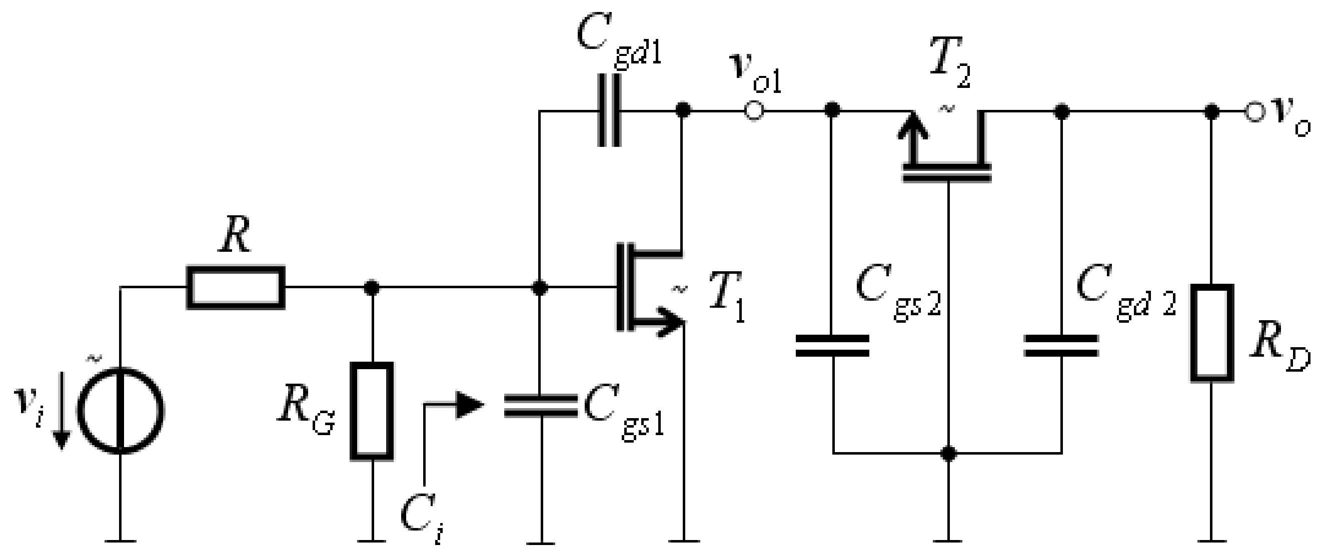
$$R_o = R_D||(r_{o1} + r_{o2} + g_{m2}r_{o1}r_{o2}) \approx R_D$$

$$g_m = g_{m1} = g_{m2}$$

$$A_v = -g_m R_D$$

OPTIONAL

➢ Cascode amplifiers – CS + CG – high frequency

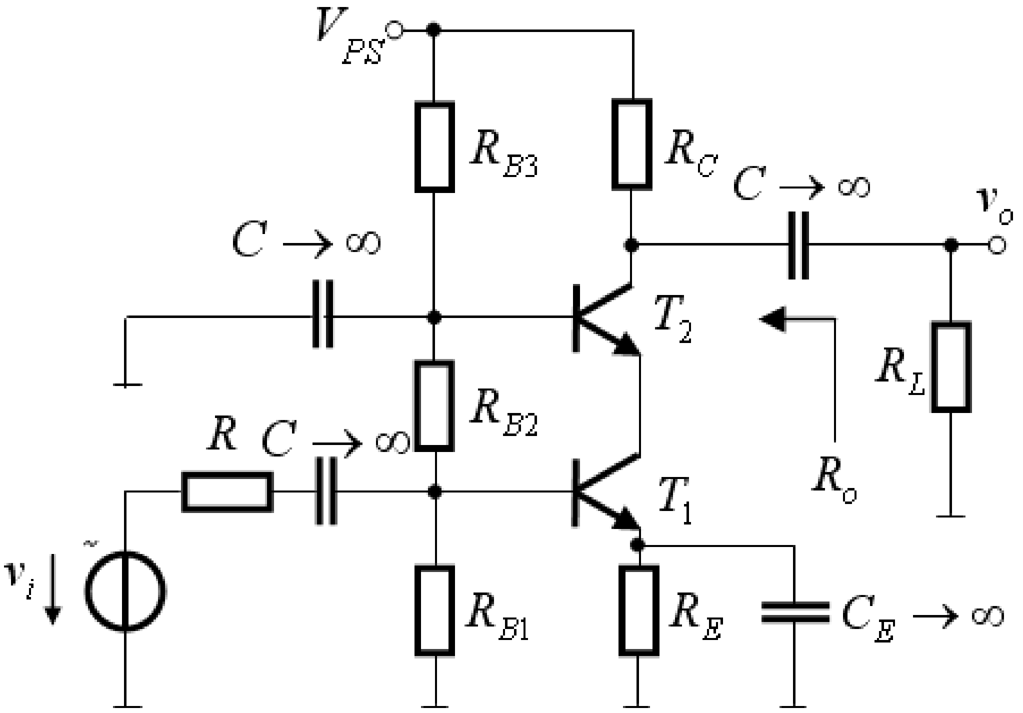


$$C_i = C_{gs1} + (1 - A_{v1})C_{gd1} = C_{gs1} + 2C_{gd1}$$

$$f_H = \frac{1}{2\pi(R||R_G)C_i}$$

OPTIONAL

➢ Cascode amplifiers – CE + CB (BJT amplifiers)



For identical T_1 and T_2 transistors

$$R_B = R_{B1}||R_{B2}$$

$$r_{be} = r_{be1} = r_{be2}$$

$$g_m = g_{m1} = g_{m2}$$

OPTIONAL

$$A_v = -\frac{R_B||r_{be}}{R + R_B||r_{be}} g_m (R_C||R_L)$$

$$f_H = \frac{1}{2\pi (r_{be}||R_B||R)(C_{be} + 2C_{bc})}$$

➢ To begin with

- What is a current source?

- a circuit where the current is independent of the load resistance $I_O \neq f(R_L)$

- the current is not influenced by the supply voltage, temperature variations or other operating conditions

$$I_O \neq f(V_{PS}),\ I_O \neq f(T)$$

- What is a current mirror? Where is it used?

- a circuit that generates dc currents in direct ratio with a reference current

The reference current is mirrored. $I_O = ratio \cdot I_{REF}$

- used to bias integrated circuits

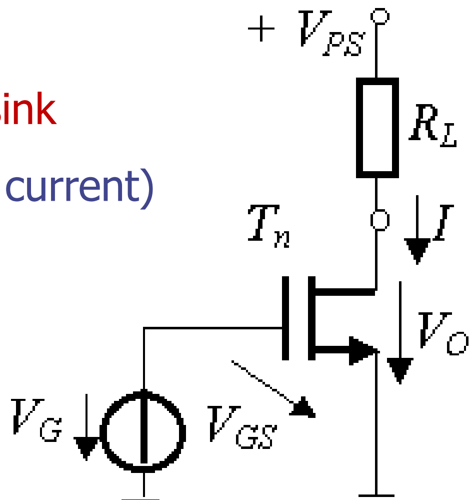
- Current sources and current mirrors w/ MOSFET or BJT?

Yes!

➢ MOSFET current sources

Current sink

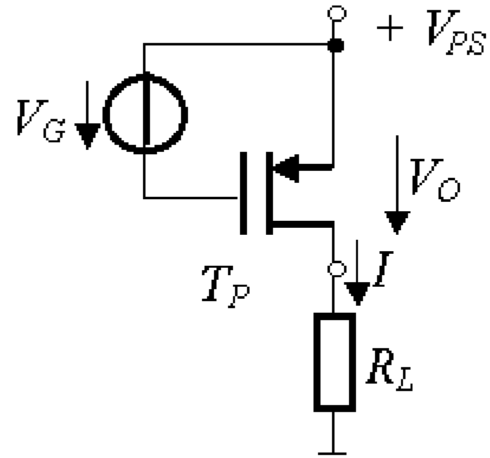
(absorbs current)



T_n – (a_F)

$$I = \beta_n(V_{GS} - V_{Thn})^2$$

$$V_O = V_{PS} - R_L I$$



Current source

(generates current)

T_p – (a_F)

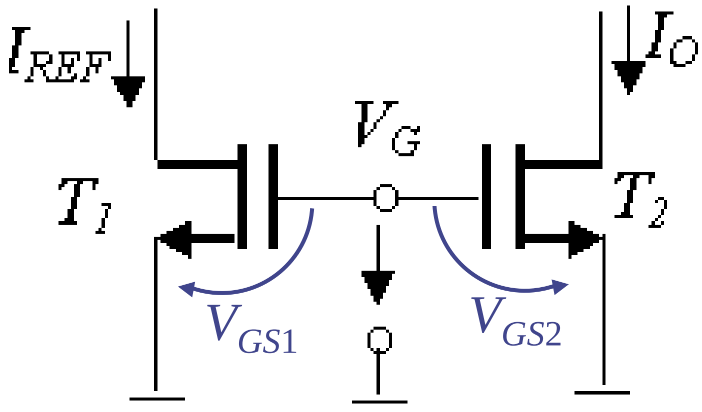
$$I = \beta_p\left(V_{GS} - V_{Thp}\right)^2$$

$V_O > V_{DSsat}$
$V_{PS} - R_L I > V_{DSsat}$
$V_{PS} - R_L I > V_{GS} - V_{Thn}$

pseudo-sources, consume power

voltage sources are necessary

➢ MOSFET current mirror (integrated transistors)



$$V_{GS1} = V_G = V_{GS2}$$

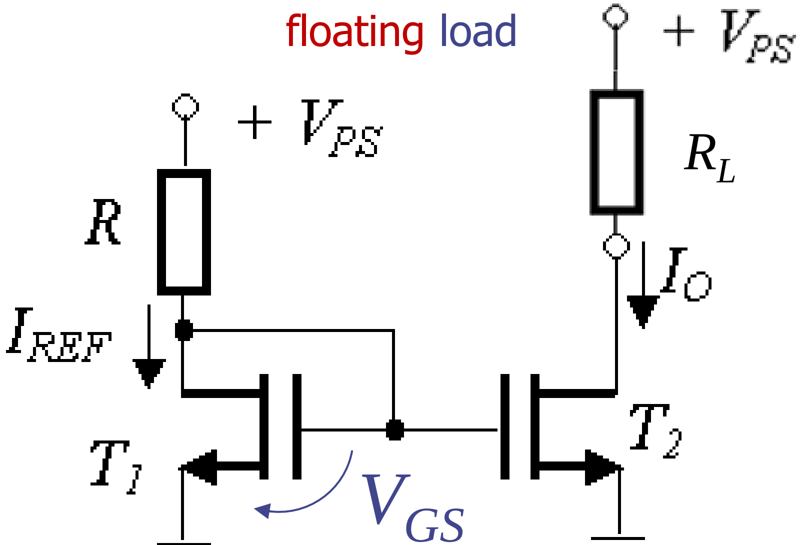
$$I = \frac{k}{2}\frac{W}{L}(V_{GS} - V_{Th})^2$$

If $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$ then $I_O = I_{REF}$

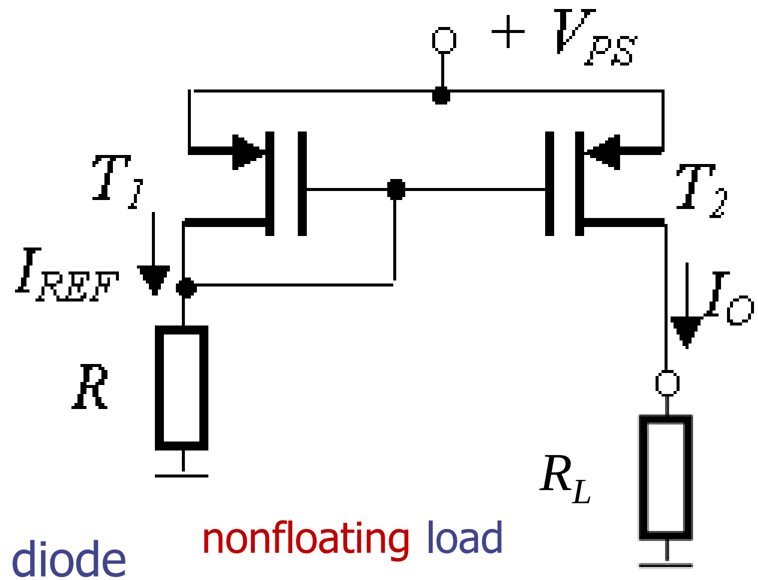
For identical transistors, the currents are equal.

How can I_{REF} be obtained?

➢ MOSFET current mirror (integrated transistors)



T_1 – MOS diode
(D connected to G)



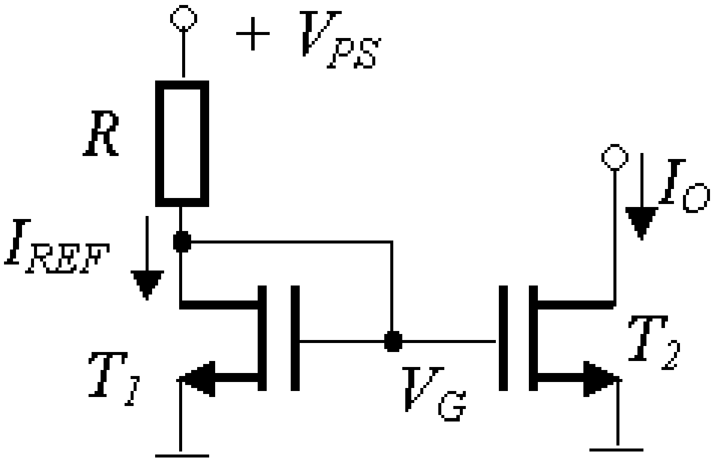
To compute I_{REF}:

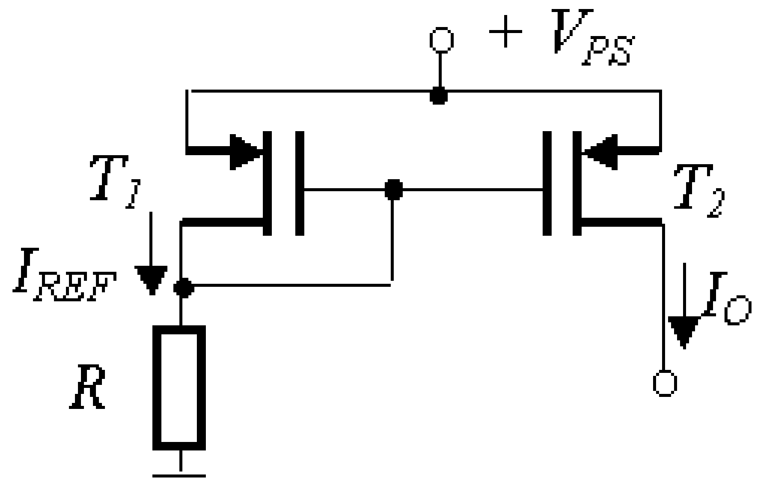
$$\begin{cases} I_{REF} = \dfrac{V_{PS} - V_{GS}}{R} \\ I_{REF} = \dfrac{K_1}{2}\left(\dfrac{W}{L}\right)_1 (V_{GS} - V_{Th1})^2 \end{cases}$$

To size R for a certain I_{REF}:

$$R = \frac{V_{PS} - V_{GS}}{I_{REF}}$$

- MOSFET current multiplier mirrors (different W/L ratios)



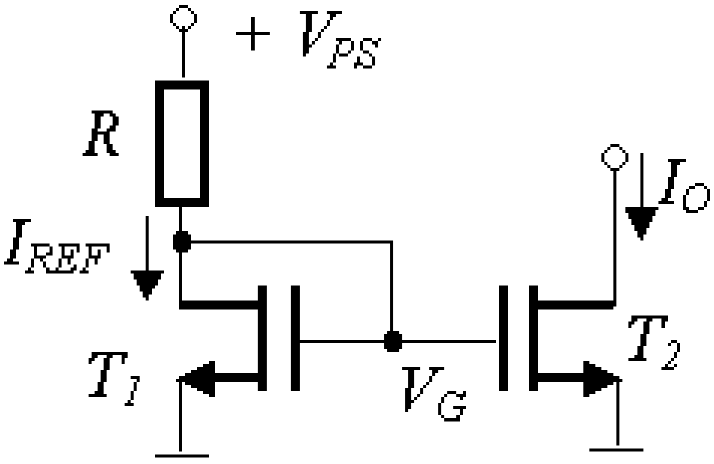


$$I_{REF} = \frac{K_1}{2}\left(\frac{W}{L}\right)_1 (V_G - V_{Th1})^2$$

$$I_O = \frac{K_2}{2}\left(\frac{W}{L}\right)_2 (V_G - V_{Th2})^2$$

If $K_1 = K_2$ and $V_{Th1} = V_{Th2}$ then $I_O = \frac{(W/L)_2}{(W/L)_1} I_{REF}$

➢ MOSFET current multiplier mirrors – example 1



V_{PS} = 12 V, K = 40 μA/V²,
V_{Th} = 1.2 V, $(W/L)_1$ = 2, $(W/L)_2$ = 8

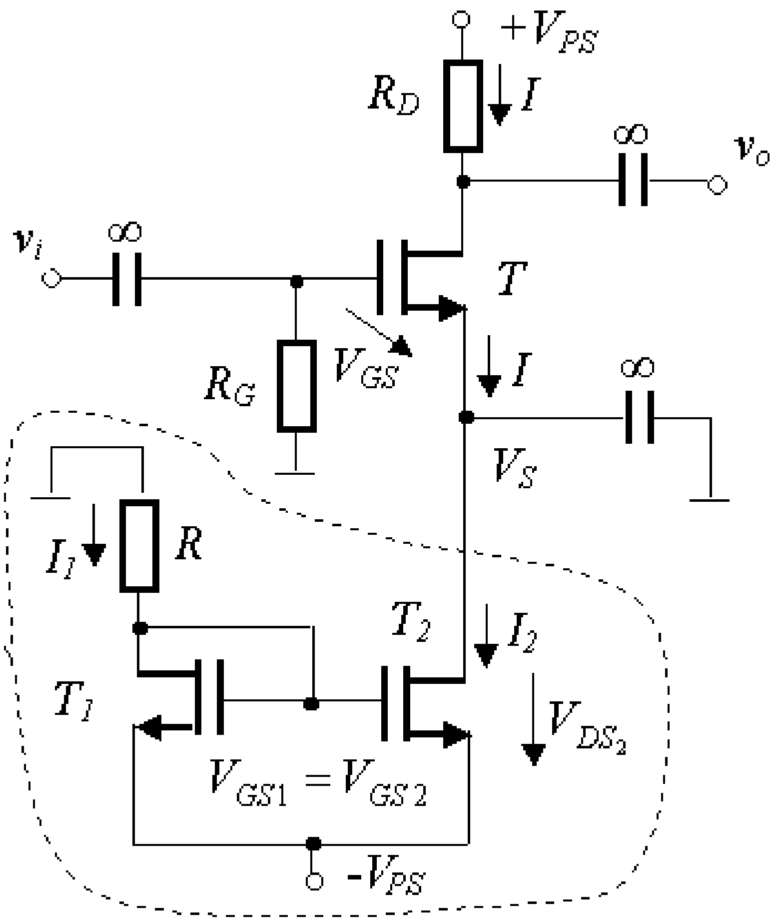
Size R for I_O = 800 μA.

$$I_{REF} = \frac{(W/L)_1}{(W/L)_2} I_O = \frac{2}{8} \cdot 800 = 200\ \mu\text{A}$$

$$V_{GS1} = V_{Th} + \sqrt{\frac{I}{\frac{K}{2}\left(\frac{W}{L}\right)_1}} = 1.2 + \sqrt{\frac{200}{\frac{40}{2} \cdot 2}} = 3.44\ \text{V}$$

$$R = \frac{V_{PS} - V_{GS1}}{I_{REF}} = \frac{12 - 3.44}{0.2} = 42.8\ \text{K}\Omega$$

➢ MOSFET current multiplier mirrors – example 2



V_{PS} = 12 V, K = 0.04 mA/V^2, R = 42.8 KΩ

V_{Th}= 1.2 V, $(W/L)_1$= 2, $(W/L)_2$ = 10, (W/L) = 4

Compute I. What is the operating region for T_1 and T_2?

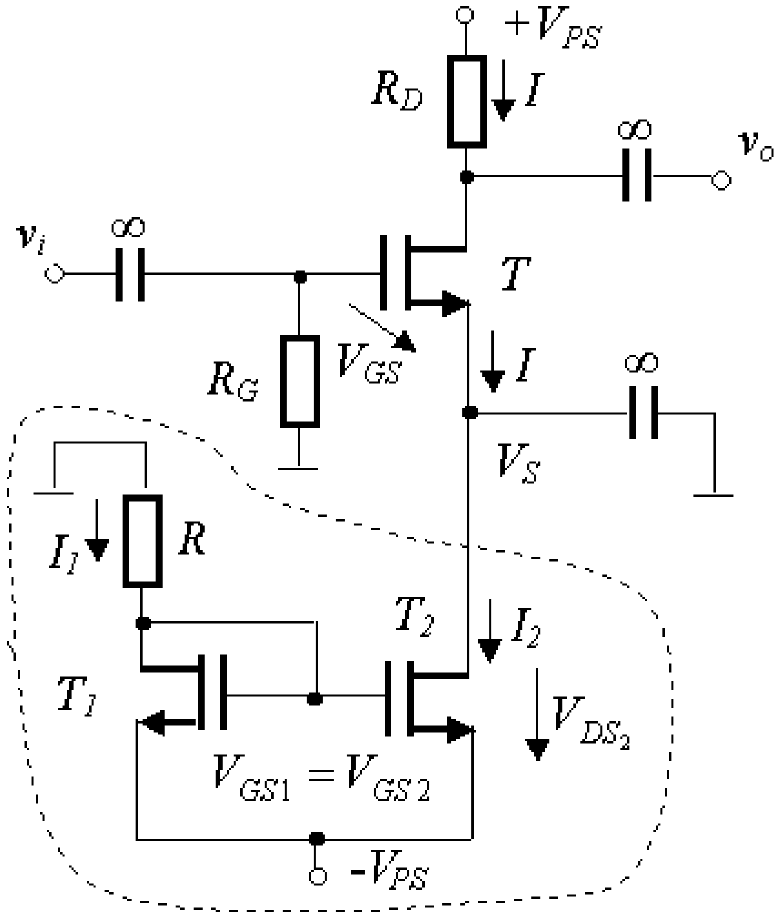
$$\begin{cases} I_1 = \dfrac{0 - V_{GS1} - (-V_{PS})}{R} = \dfrac{V_{PS} - V_{GS1}}{R} \\ I_1 = \dfrac{K}{2}\left(\dfrac{W}{L}\right)_1 (V_{GS1} - V_{Th})^2; \end{cases}$$

$$I_1 = 0.2 \text{ mA}$$

$$I = I_2 = I_1 \frac{(W/L)_2}{(W/L)_1}$$

$$I = 0.2 \cdot \frac{10}{2} = 1 \text{ mA}$$

MOSFET current multiplier mirrors – example 2



V_{PS} = 12 V, K = 0.04 mA/V^2, R = 42.8 KΩ

V_{Th} = 1.2 V, $(W/L)_1$ = 2, $(W/L)_2$ = 10, (W/L) = 4

Compute I. What is the operating region for T_1 and T_2?

T_1 - MOS diode, always in (a_F)

$$I = \frac{K}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{Th})^2$$

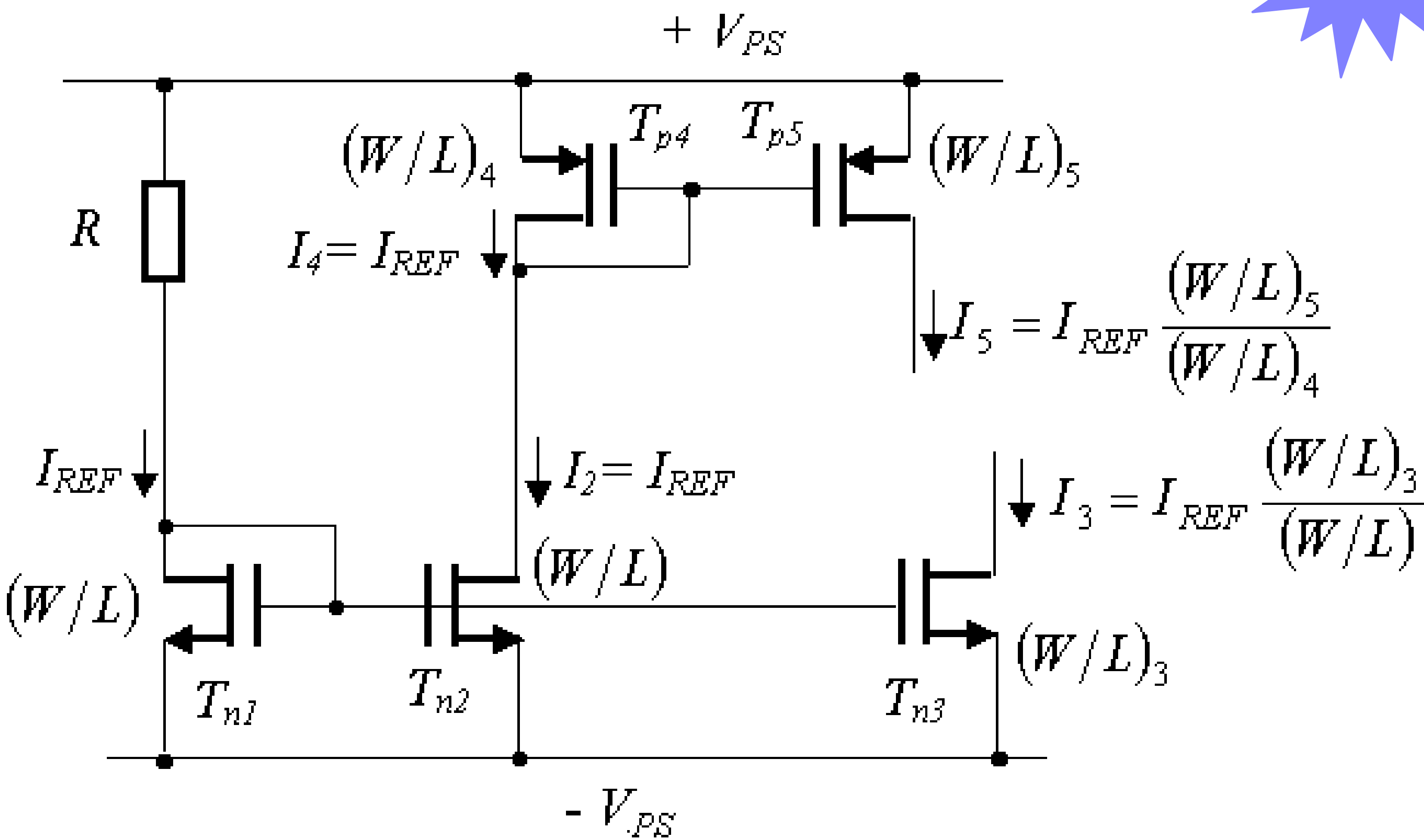
$$V_{GS} = V_{Th} + \sqrt{\frac{I}{\frac{K}{2}\left(\frac{W}{L}\right)}} = 1.2 + \sqrt{\frac{1}{\frac{0.04}{2} \cdot 4}} \approx 4.7 \text{ V}$$

$V_S = -4.7$ V $\quad V_{DS_2} = -4.7 - (-12) = 7.3$ V

$V_{GS_2} \approx 3.4$ V $\quad V_{DSsat2} = 2.2$ V, $T_2 - (a_F)$

➢ Biasing a MOSFET integrated circuit

OPTIONAL

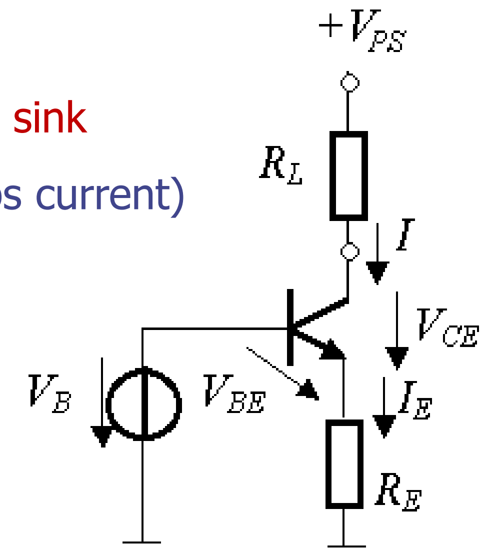


➢ BJT current sources

T must stay in a_F: $V_{CE} > V_{CEsat}$

Current sink

(absorbs current)



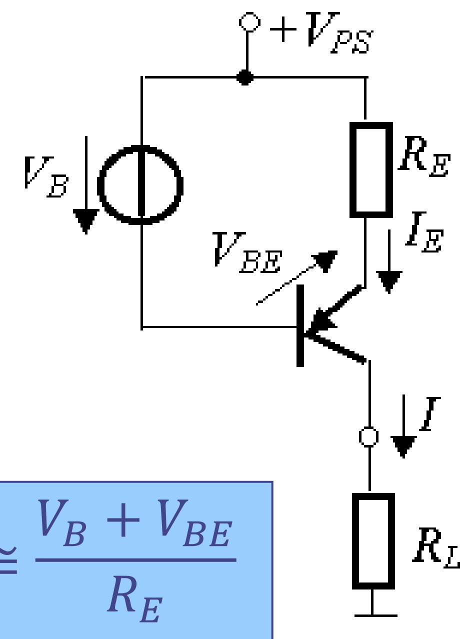
$$R_L I < V_{PS} - V_{CEsat} - V_B + V_{BE}$$

$$I = \frac{\beta}{\beta + 1} I_E; \quad I_E = \frac{V_B - V_{BE}}{R_E}$$

$$I = \frac{\beta}{\beta + 1} \frac{V_B - V_{BE}}{R_E} \cong \frac{V_B - V_{BE}}{R_E}$$

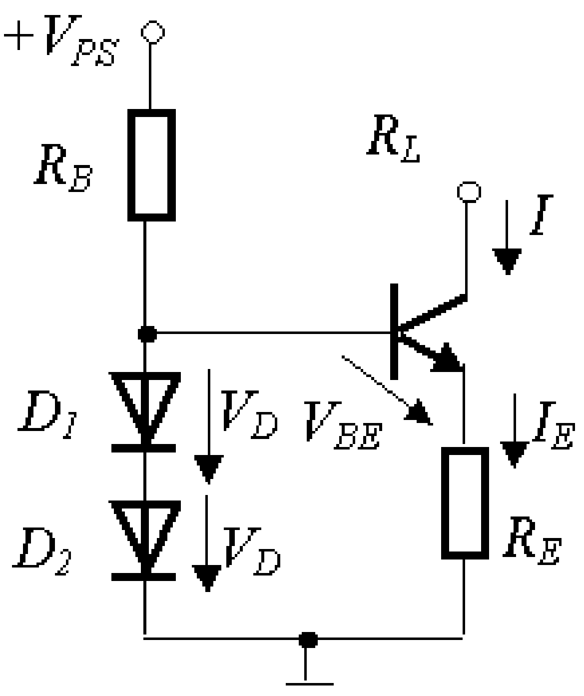
Current source

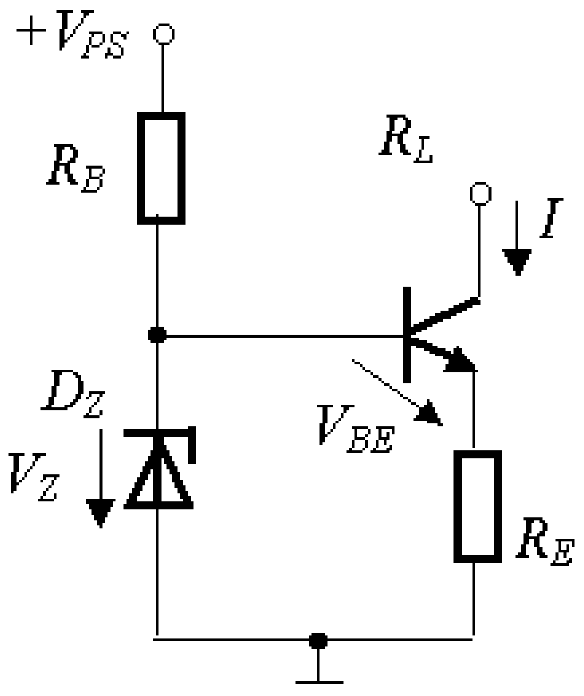
(generates current)



$$I \cong \frac{V_B + V_{BE}}{R_E}$$

➢ BJT current sinks – setting the base potential

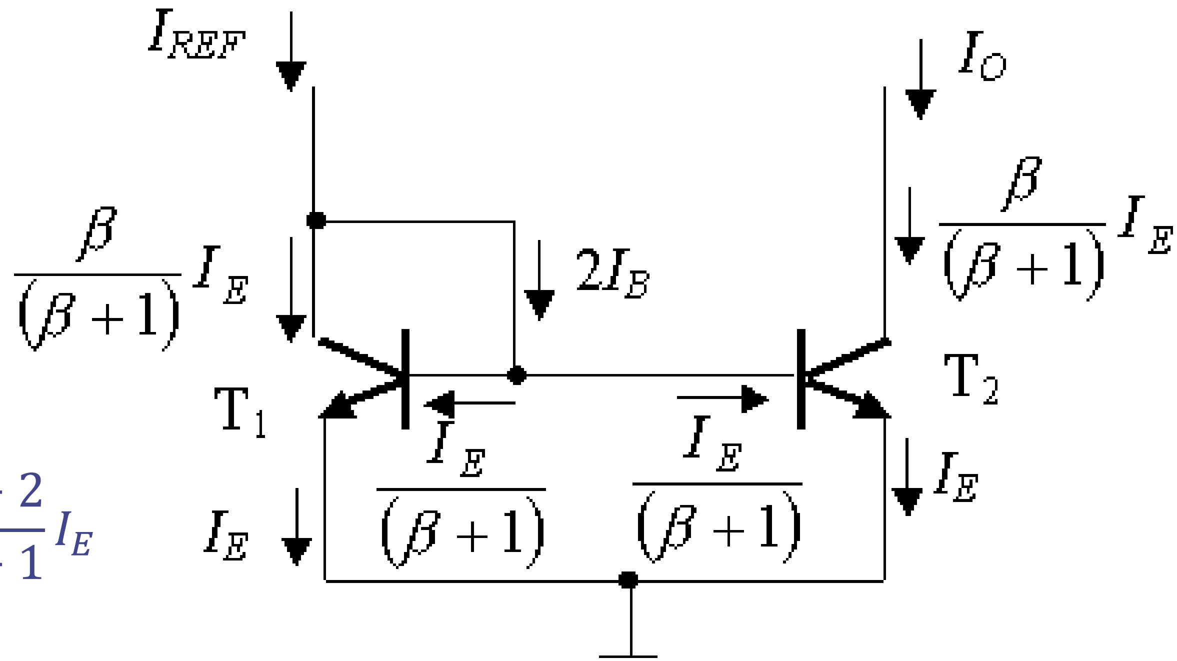




$$I = \frac{2V_D - V_{BE}}{R_E} \cong \frac{0.7\ \text{V}}{R_E}$$

$$I = \frac{V_Z - V_{BE}}{R_E}$$

➢ BJT current mirror

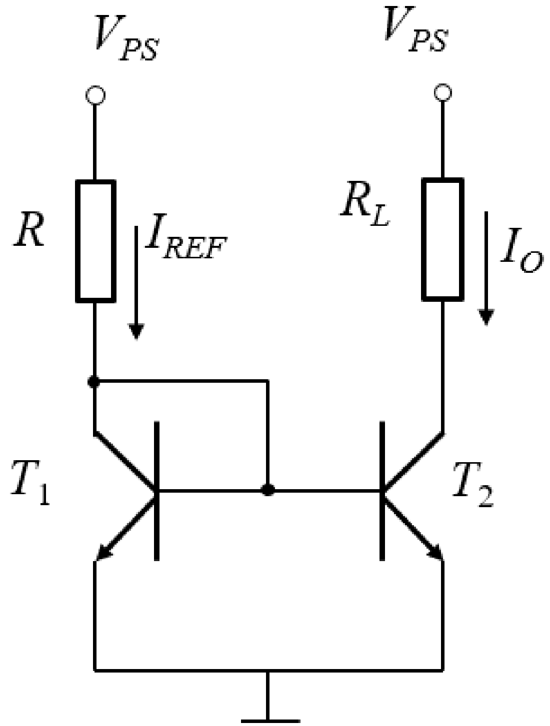


$$I_{REF} = \frac{\beta}{\beta+1} I_E + 2\frac{I_E}{\beta+1} = \frac{\beta+2}{\beta+1} I_E$$

$$I_O = \frac{\beta}{\beta+1} I_E$$

$$I_O = \frac{1}{1+2/\beta} I_{REF} \cong I_{REF}$$

➢ BJT current mirror

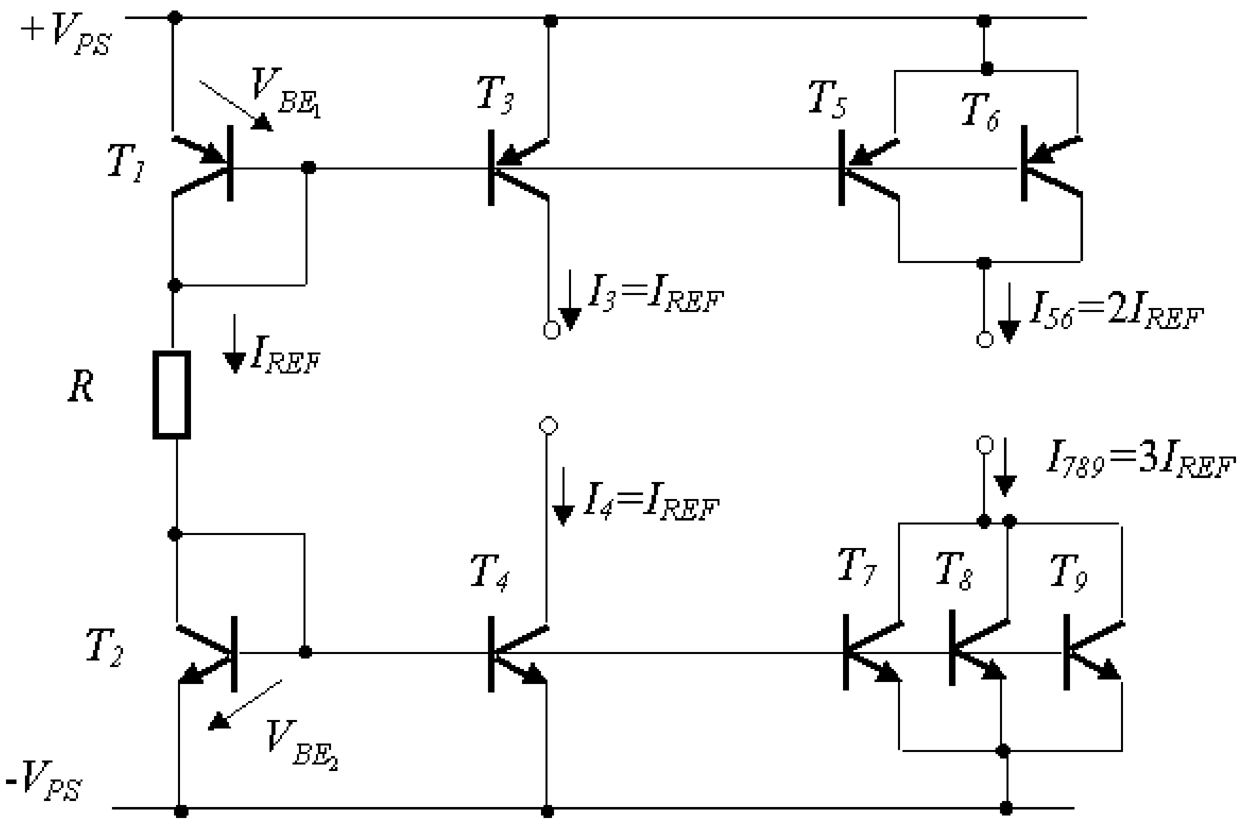


$$I_O = \frac{1}{1 + 2/\beta} I_{REF} \cong I_{REF}$$

$$I_{REF} = \frac{V_{PS} - V_{BE}}{R} = \frac{V_{PS} - 0.7}{R}$$

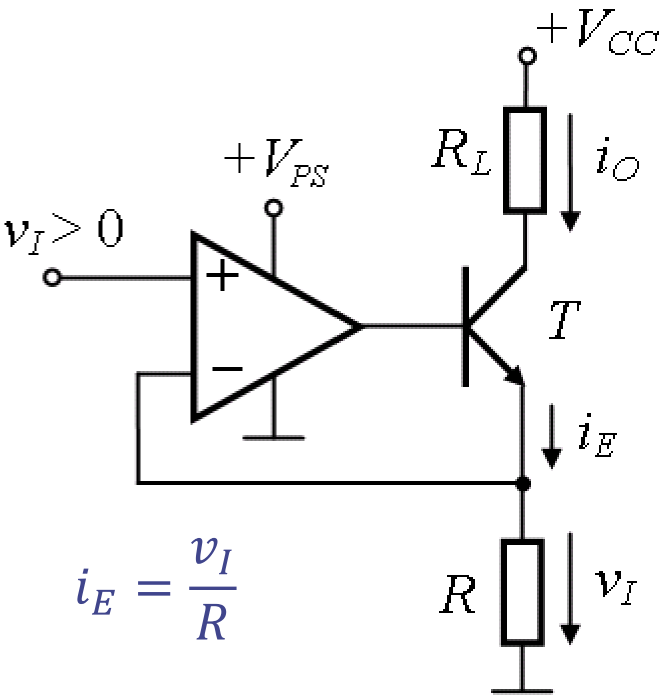
OPTIONAL

➢ Biasing a BJT integrated circuit



$$I_{REF} = \frac{2V_{PS} - V_{BE_1} - V_{BE_2}}{R}$$

➢ Current sources w/ OpAmp and T, floating load

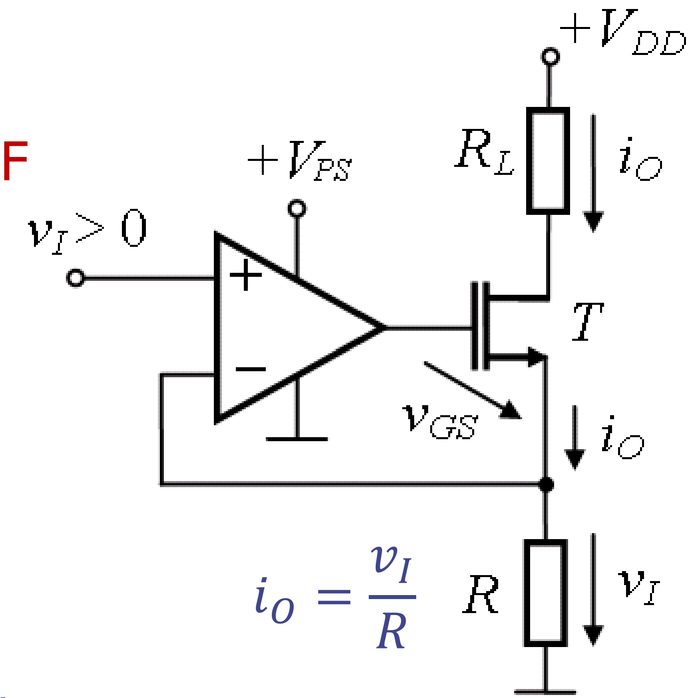


$$i_E = \frac{v_I}{R}$$

$$i_O = \frac{\beta}{\beta + 1}\frac{v_I}{R} \approx \frac{v_I}{R}$$

$$R_L i_O < V_{CC} - v_I - V_{CEsat}$$

Negative feedback NF



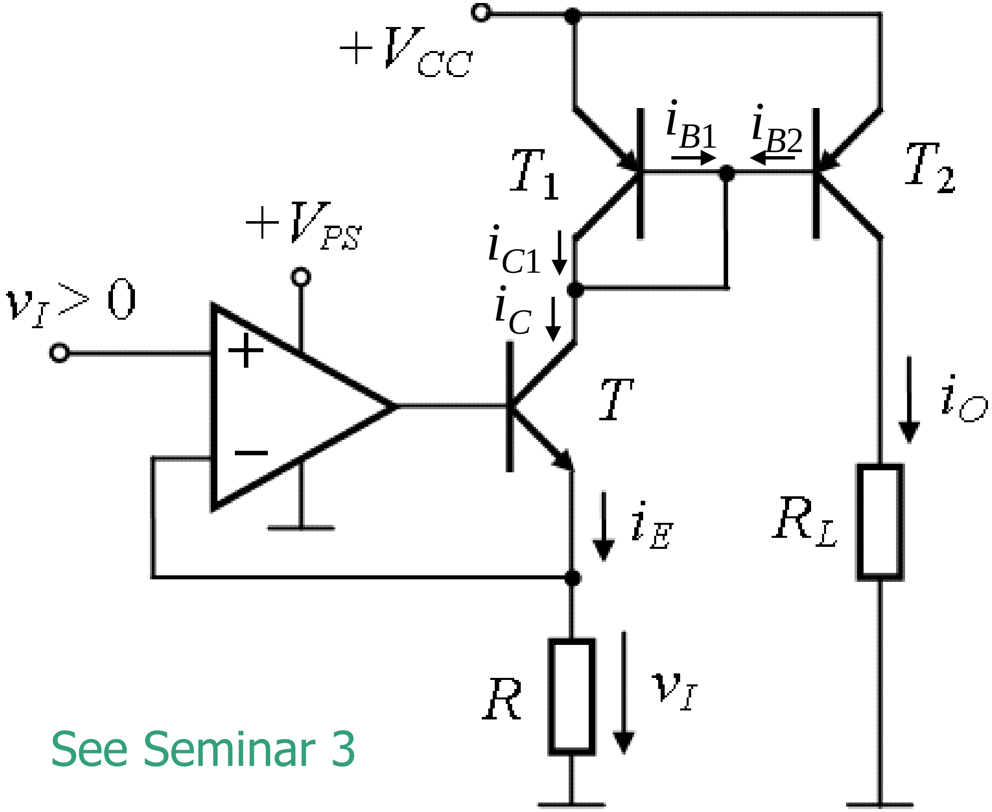
$$i_O = \frac{v_I}{R}$$

Adjustable current sources:

- modifying v_I - voltage-controlled current source
- a potentiometer in series with *R*.

See Seminar 3 & Lab 6

➢ Current source w/ OpAmp and T, nonfloating load



See Seminar 3

$$i_O = \frac{\beta}{\beta+1}\frac{1}{1+\frac{2}{\beta_1}}\frac{v_I}{R} \approx \frac{v_I}{R}$$

T_1 and T_2 - identical transistors

$$i_{B1} = i_{B2} \qquad i_O = \beta_1 i_{B2}$$

$$i_C = i_{C1} + 2i_{B2} = \beta_1 i_{B1} + 2i_{B2}$$

$$i_C = i_{B2}(\beta_1 + 2) \quad i_{B2} = \frac{i_C}{\beta_1 + 2}$$

$$i_O = \beta_1 \frac{i_C}{\beta_1 + 2} = i_C \frac{\beta_1}{\beta_1 + 2}$$

$$i_O = i_C \frac{1}{1+\frac{2}{\beta_1}} \qquad i_C = \frac{\beta}{\beta+1} i_E$$

$$i_O = \frac{\beta}{\beta+1}\frac{1}{1+\frac{2}{\beta_1}} i_E \quad i_E = \frac{v_I}{R}$$

Summary

- Frequency response of transistor amplifiers
- Current sources
- Current mirrors

Next week: Power amplifiers – class A, B, AB