Sinusoidal Oscillators

- Signal generators: sinusoidal, rectangular, triangular, sawtooth, etc.

  - Sine wave generation: frequency selective network in a feedback loop of a PF amplifier: ***sinusoidal oscillator***

-Oscillation frequency: f_0

-Oscillation amplitude: $\hat{V}_o$

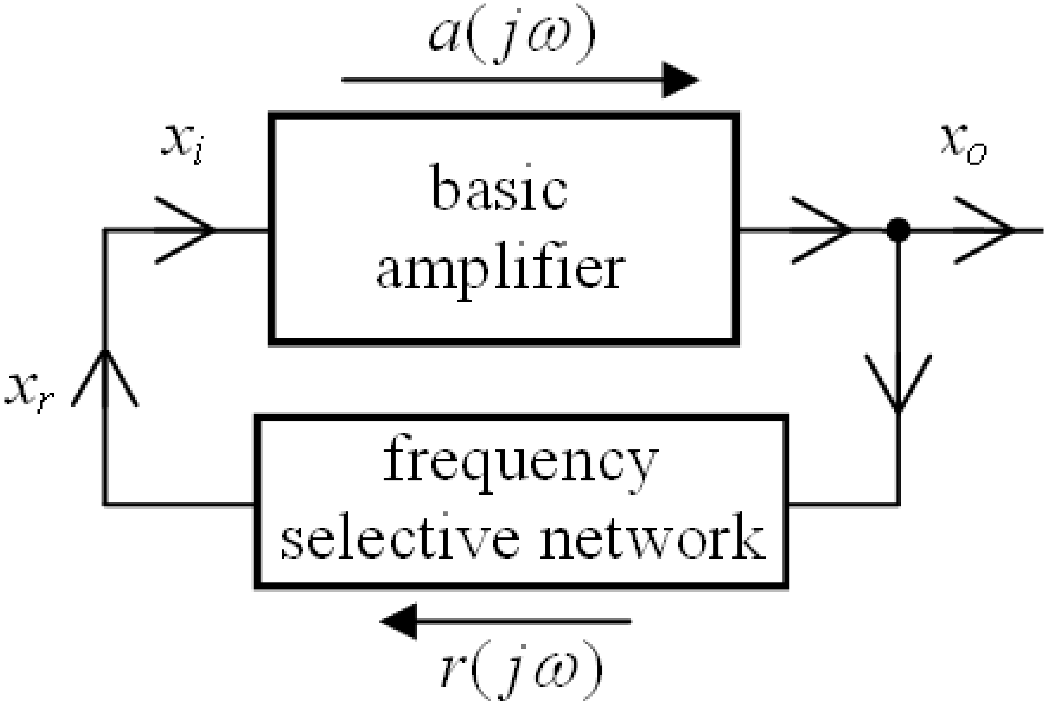
- Oscillation criterion

- Frequency stability

- Amplitude stability

- Distortion coefficient

Oscillator feedback loop



In the complex domain

For a unique *f*$_0$

$$\omega_0 = 2\pi f_0$$

Signal reconstruction on the feedback loop

Frequency dependent components (*C*, *L*)

$$\boxed{a(j\omega_0)r(j\omega_0) = 1}$$

Barkhausen criterion

$$Zc = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi f C}$$

$$Z_L = j\omega L = j2\pi f L$$

Complex number - short review

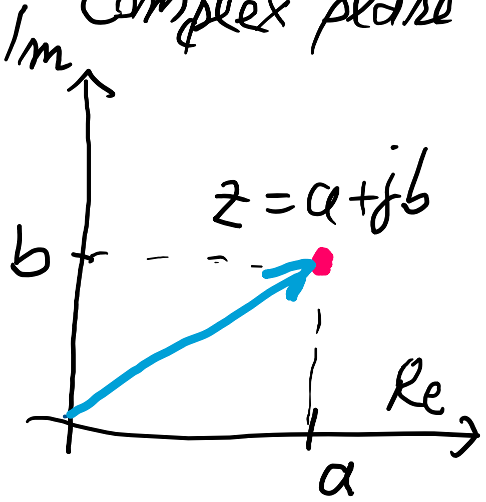
$z = a + jb$

$$\begin{cases} Re(z) = a \\ Im(z) = b \end{cases}$$

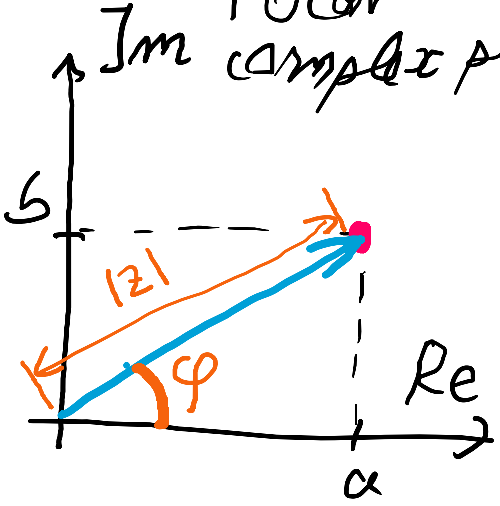
$j = \sqrt{-1}$ imaginary unit

imaginary number

Cartezian complex plane



Polar complex plane



- absolute value, modulus; $|z| = \sqrt{a^2 + b^2}$ magnitude
- argument; phase $\varphi = \tan^{-1}\left(\frac{b}{a}\right)$

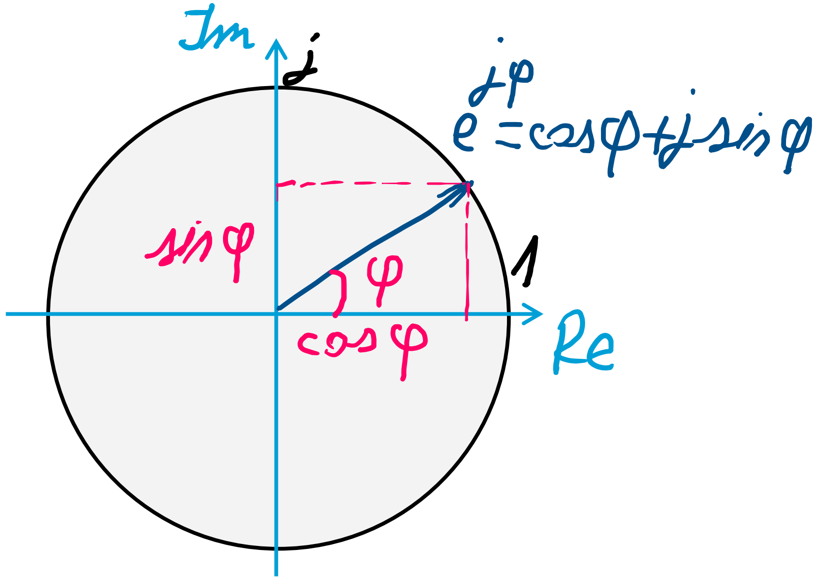
trigonometric form:

$z = |z| \cdot (\cos\varphi + j\sin\varphi)$

Euler's formula

$z = |z| e^{j\varphi}$

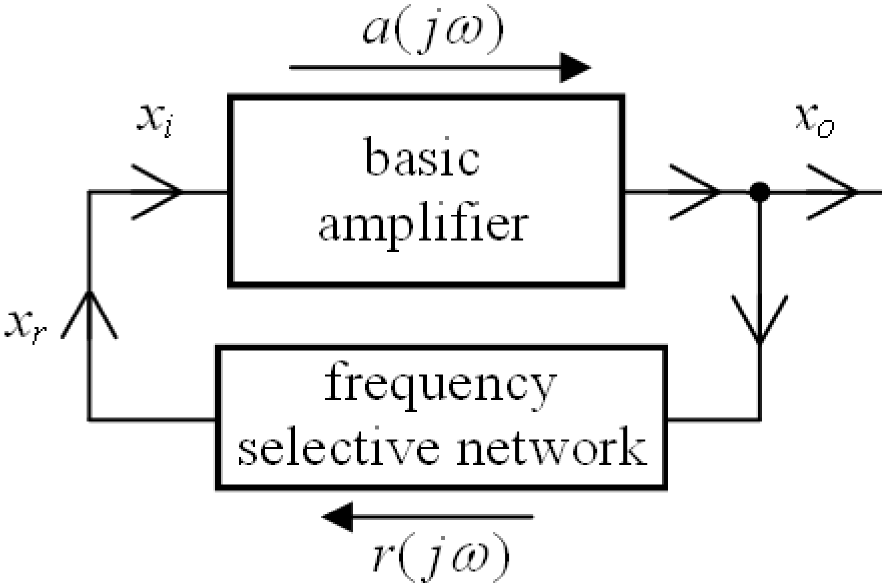
Unit circle



Oscillation criteria

$$a(j\omega) = |a(j\omega)| e^{j\varphi_a}$$

$$r(j\omega) = |r(j\omega)| e^{j\varphi_r}$$



$$a(j\omega_0) r(j\omega_0) = |a(j\omega_0)||r(j\omega_0)| e^{j(\varphi_a + \varphi_r)} = 1$$

- ✓ magnitude condition: $\boxed{|a(j\omega_0)||r(j\omega_0)| = 1}$ gives a_0

  The loop gain is equal to unity in magnitude

- ✓ phase condition: $\boxed{\varphi_a + \varphi_r = 2k\pi}$ gives f_0

  The phase shift around the loop is zero or an integer multiple of 2π

RC Oscillators

- **Basic amplifier** – frequency independent
  - inverting $\varphi_a = 180^{\mathrm{o}}$
  - noninverting $\varphi_a = 0$

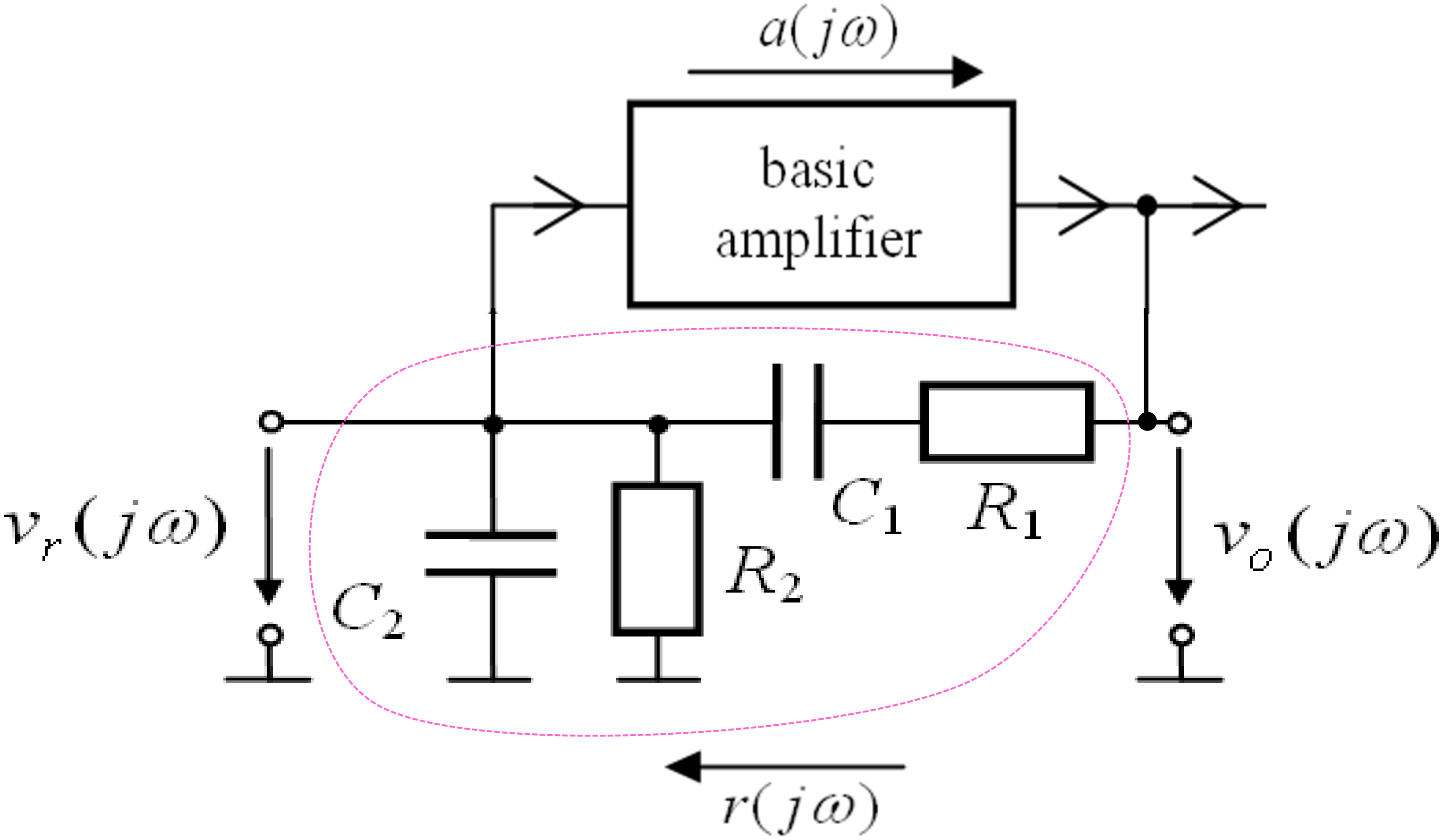
- Frequency selective **feedback network**

To fulfil the phase condition, there must be **a unique frequency, f_0** where the phase shift is:

$$\varphi_r = 180^0, \quad \text{if} \quad \varphi_a = 180^0$$

$$\varphi_r = 0^0, \quad \text{if} \quad \varphi_a = 0^0$$

RC Oscillators

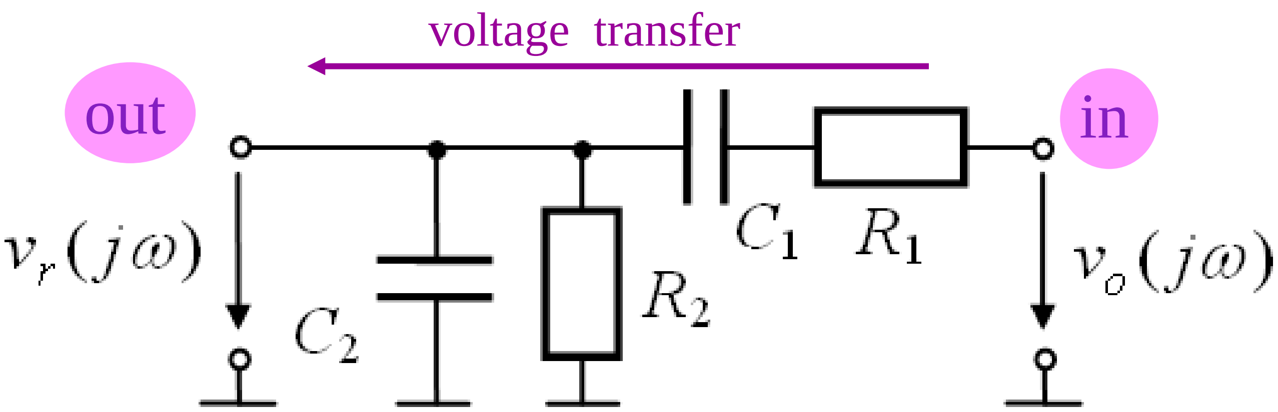


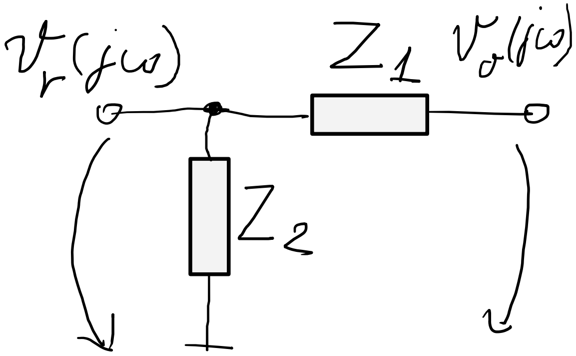
Feedback network

Frequency selective network

WIEN Bridge

WIEN Bridge Frequency selective network **Transfer function**





$$v_r(j\omega)=r(j\omega)v_o(j\omega)$$

$$\boxed{r(j\omega)=\frac{v_r(j\omega)}{v_o(j\omega)}}$$

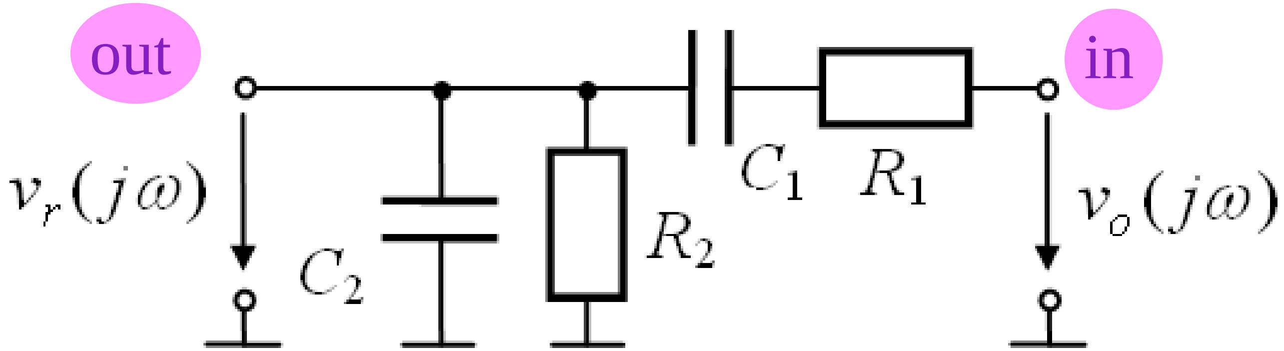
$$r(j\omega)=\frac{Z_2}{Z_1+Z_2}$$

$$Z_1=R_1+\frac{1}{j\omega C_1}$$

$$Z_2=R_2\parallel\frac{1}{j\omega C_2}$$

$$\boxed{r(j\omega)=\frac{v_r(j\omega)}{v_o(j\omega)}=\frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}+j\left(\omega R_1C_2-\frac{1}{\omega R_2C_1}\right)}}$$

complex number



WIEN Bridge

$$\boxed{r(j\omega)=\frac{v_r(j\omega)}{v_o(j\omega)}=\frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}+j\left(\omega R_1C_2-\frac{1}{\omega R_2C_1}\right)}}$$

$$r(j\omega)=\left|r(j\omega)\right|e^{j\varphi_r}$$

$$\left|r(j\omega)\right|=\frac{1}{\sqrt{\left(1+\frac{R_1}{R_2}+\frac{C_2}{C_1}\right)^2+\left(\omega R_1C_2-\frac{1}{\omega R_2C_1}\right)^2}}$$

modulus

$$\varphi_r=-arctg\left(\frac{\omega R_1C_2-\frac{1}{\omega R_2C_1}}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}}\right)$$

phase

Modulus

WIEN Bridge

$$\left|r(j\omega)\right| = \frac{1}{\sqrt{\left(1+\frac{R_1}{R_2}+\frac{C_2}{C_1}\right)^2+\left(\omega R_1 C_2-\frac{1}{\omega R_2 C_1}\right)^2}}$$

$\omega \to 0$ $\left|r(j\omega)\right| \to 0$ asymptote

$\omega \to \infty$ $\left|r(j\omega)\right| \to 0$ asymptote

The maximum value (as a function of ω):

$$\omega = \omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \qquad \left|r(j\omega_0)\right| = \frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}}$$

Modules

WIEN Bridge

$$\left|r(j\omega)\right| = \frac{1}{\sqrt{\left(1+\frac{R_1}{R_2}+\frac{C_2}{C_1}\right)^2+\left(\omega R_1 C_2-\frac{1}{\omega R_2 C_1}\right)^2}}$$

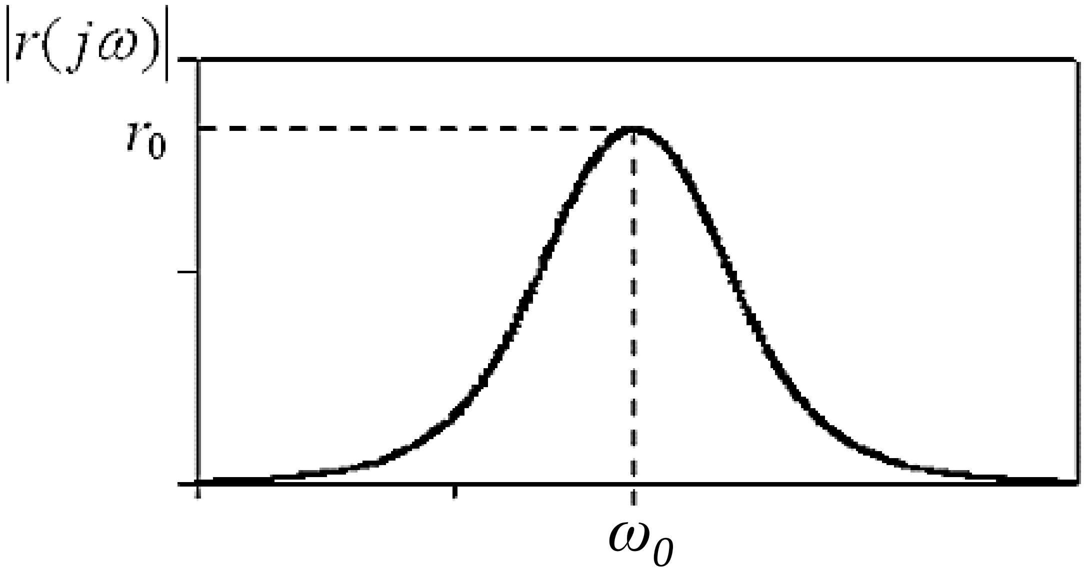
$\omega \to 0 \quad \left|r(j\omega)\right| \to 0$ asymptote

$\omega \to \infty \quad \left|r(j\omega)\right| \to 0$ asymptote

The maximum value:

$$\omega = \omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$r_0 = \left|r(j\omega_0)\right| = \frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}}$$



Phase

WIEN Bridge

$$\varphi_r = -arctg\left(\frac{\omega R_1 C_2 - \frac{1}{\omega R_2 C_1}}{1 + \frac{R_1}{R_2} + \frac{C_2}{C_1}} \right)$$

$\omega \to 0$ $\varphi_r \to +90^0$ asymptote

$\omega \to \infty$ $\varphi_r \to -90^0$ asymptote

$\omega = \omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$ $\varphi_r = 0^0$ intermediate value

Phase

WIEN Bridge

$$\varphi_r = -arctg\left(\frac{\omega R_1 C_2 - \dfrac{1}{\omega R_2 C_1}}{1 + \dfrac{R_1}{R_2} + \dfrac{C_2}{C_1}}\right)$$

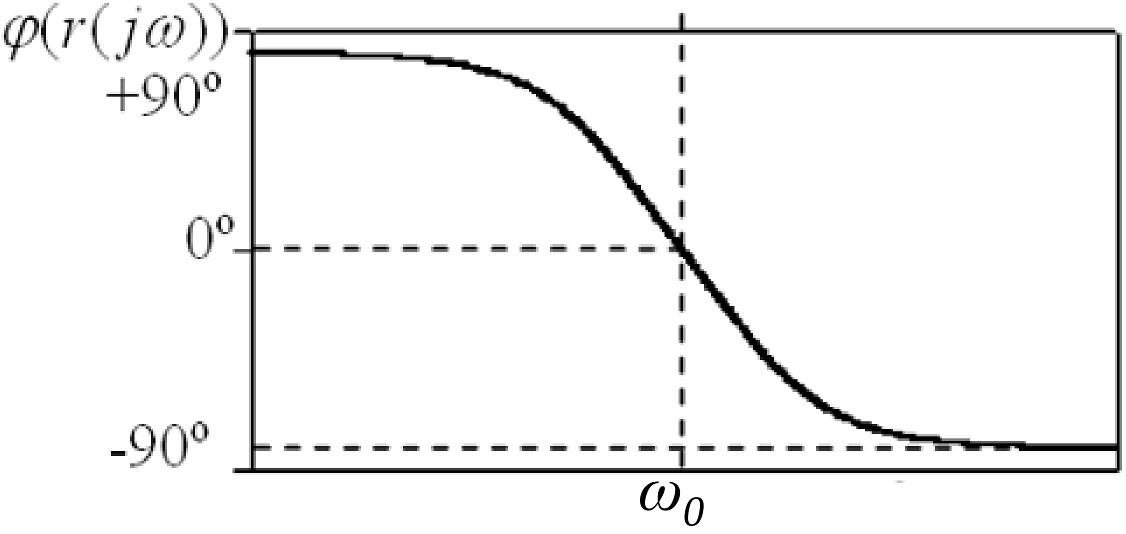
$\omega \to 0$ $\varphi_r \to +90^0$ asymptote

$\omega \to \infty$ $\varphi_r \to -90^0$ asymptote

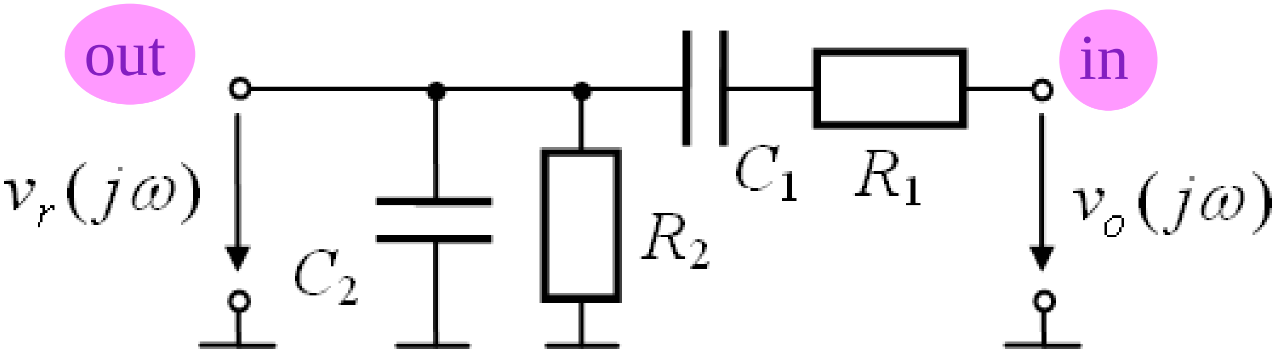
intermediate value

$$\varphi_r = 0^0$$

$$\omega = \omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$



WIEN Bridge



Frequency response

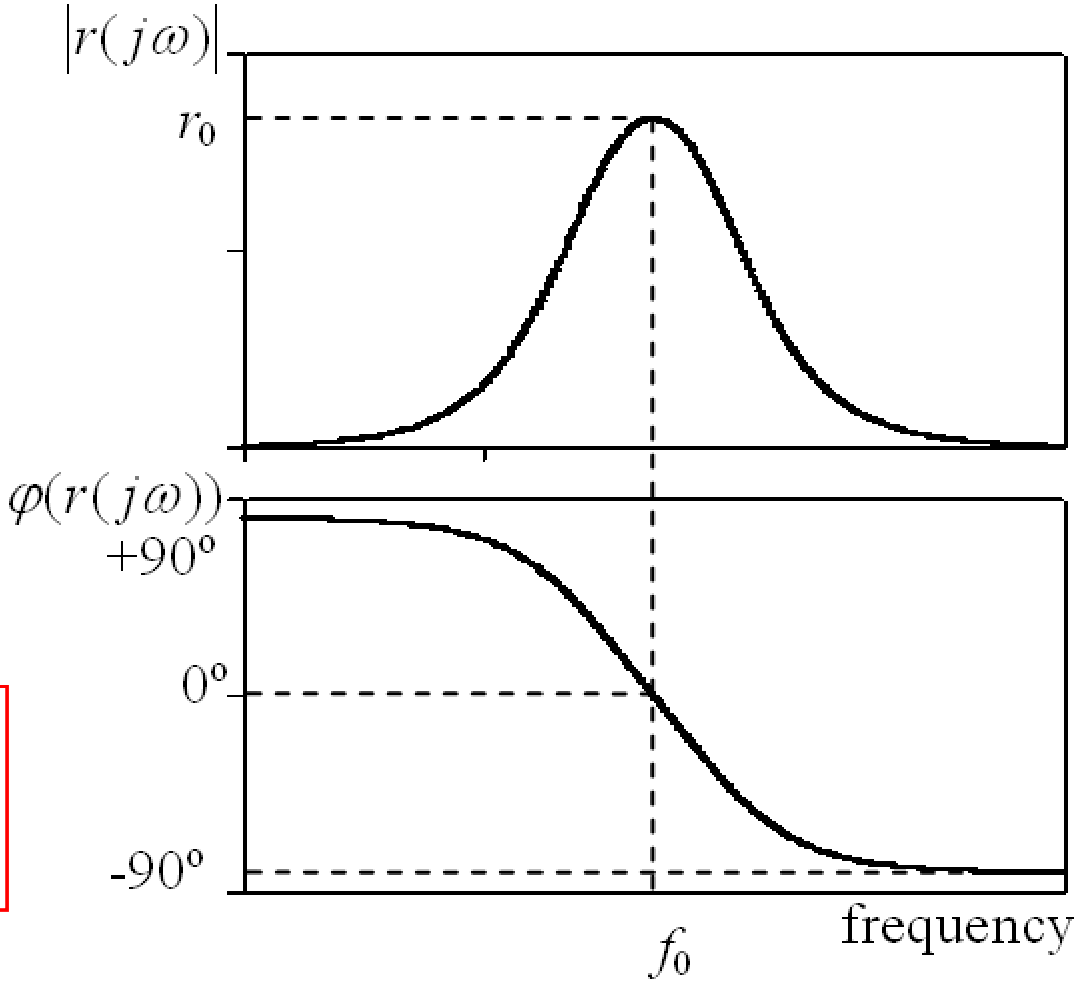
For only one **unique frequency**, f_0 we have

$$\varphi_r = 0$$

For the phase condition: noninverting amplifier

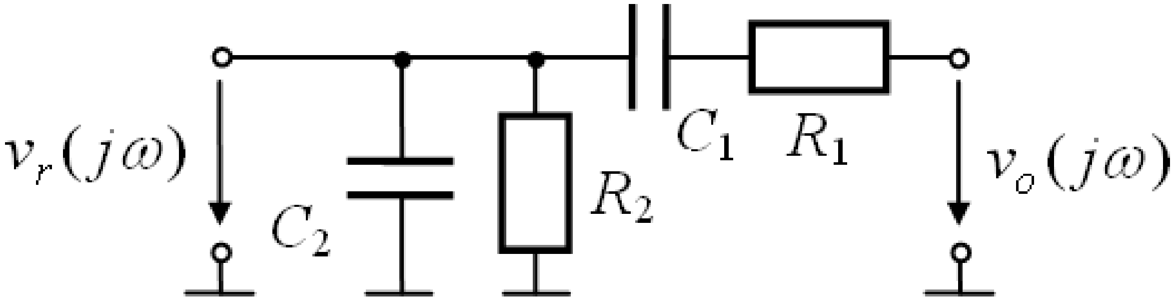
$$\varphi_a = 0$$

$$\boxed{f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}}$$



WIEN Bridge

Summary



$$r(j\omega)=\frac{v_r(j\omega)}{v_o(j\omega)}=\frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}+j\left(\omega R_1C_2-\frac{1}{\omega R_2C_1}\right)}$$

Barkhausen criterion: $a(j\omega_0)r(j\omega_0)=1$

Real number

Real number

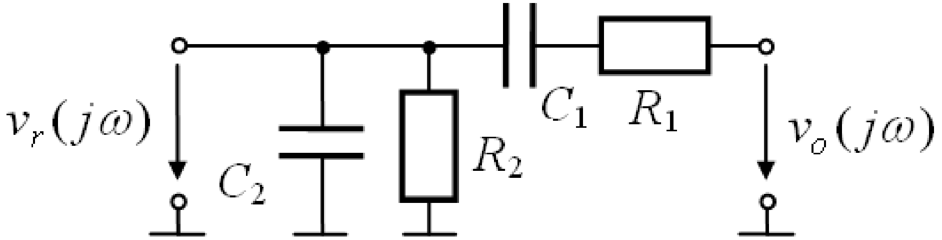
$$\omega_0R_1C_2-\frac{1}{\omega_0R_2C_1}=0$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\boxed{f_0=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}}$$

WIEN Bridge

Summary



$$r(j\omega)=\frac{v_r(j\omega)}{v_o(j\omega)}=\frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}+j\left(\omega R_1C_2-\frac{1}{\omega R_2C_1}\right)}$$

$$\boxed{f_0=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}}$$

$$\boxed{\varphi_r(j\omega_0)=0}$$

$$\boxed{\left|r(j\omega_0)\right|=\frac{1}{1+\frac{R_1}{R_2}+\frac{C_2}{C_1}}}$$

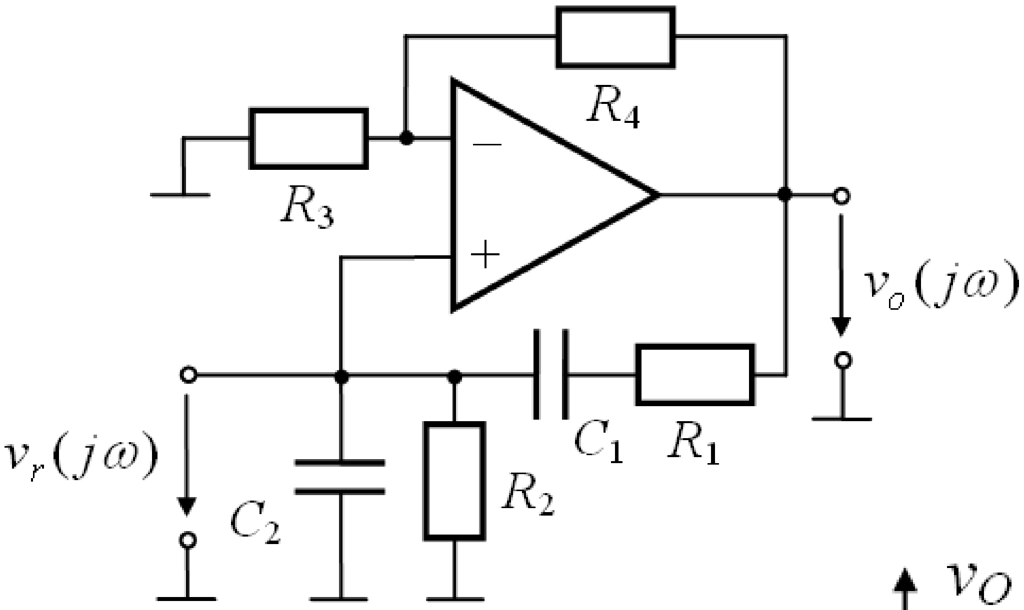
If $R_1=R_2=R$

$C_1=C_2=C$

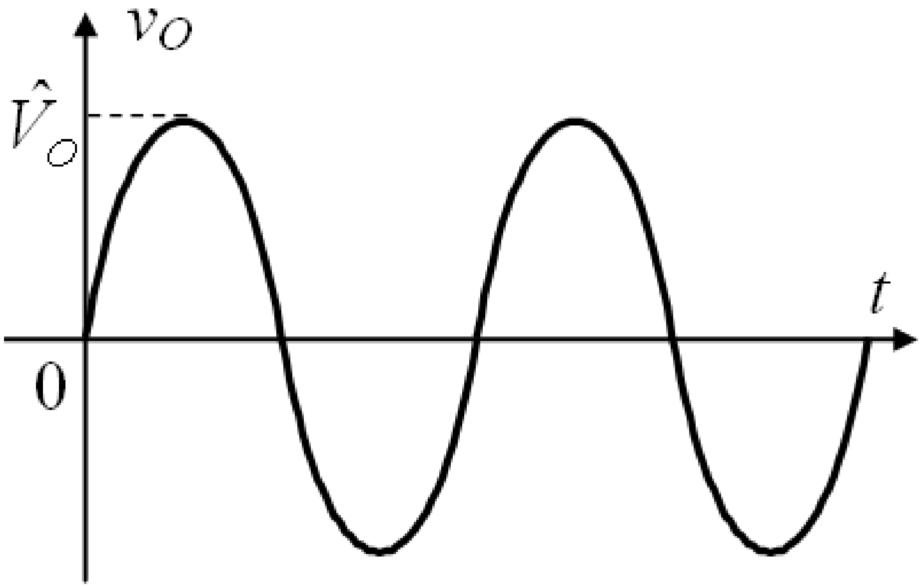
$$\boxed{f_0=\frac{1}{2\pi RC}}$$

$$\boxed{\left|r(j\omega_0)\right|=\frac{1}{3}}$$

Op amp and WIEN bridge oscillator



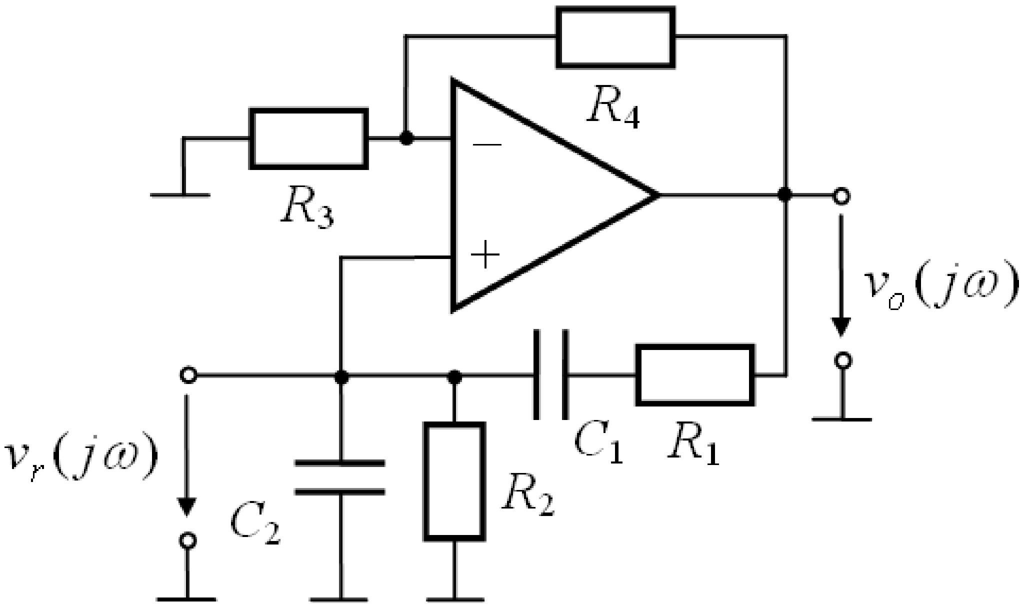
$$v_o(t) = \hat{V}_o \sin 2\pi f_0 t$$



$$\boxed{f_0 = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}}$$

$$\boxed{\left|r(j\omega_0)\right| = \frac{1}{1 + \frac{R_1}{R_2} + \frac{C_2}{C_1}}}$$

Op amp and WIEN bridge oscillator



$$\boxed{f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}}$$

$$\boxed{\left|r(j\omega_0)\right| = \frac{1}{1 + \dfrac{R_1}{R_2} + \dfrac{C_2}{C_1}}}$$

For $\begin{cases} C_1 = C_2 = C \\ R_1 = R_2 = R \end{cases}$

$$\boxed{f_0 = \frac{1}{2\pi RC}}$$

$$\boxed{\left|r(j\omega_0)\right| = \frac{1}{3}}$$

$$\left|a(j\omega_0)\right| = \frac{1}{\left|r(j\omega_0)\right|} = 3 \qquad a = 1 + \frac{R_4}{R_3} \qquad 1 + \frac{R_4}{R_3} = 3 \qquad R_4 = 2R_3$$

$$\hat{V}_o = ?$$

Nonlinearity of the gain, close to saturation

Automatic gain control (AGC)

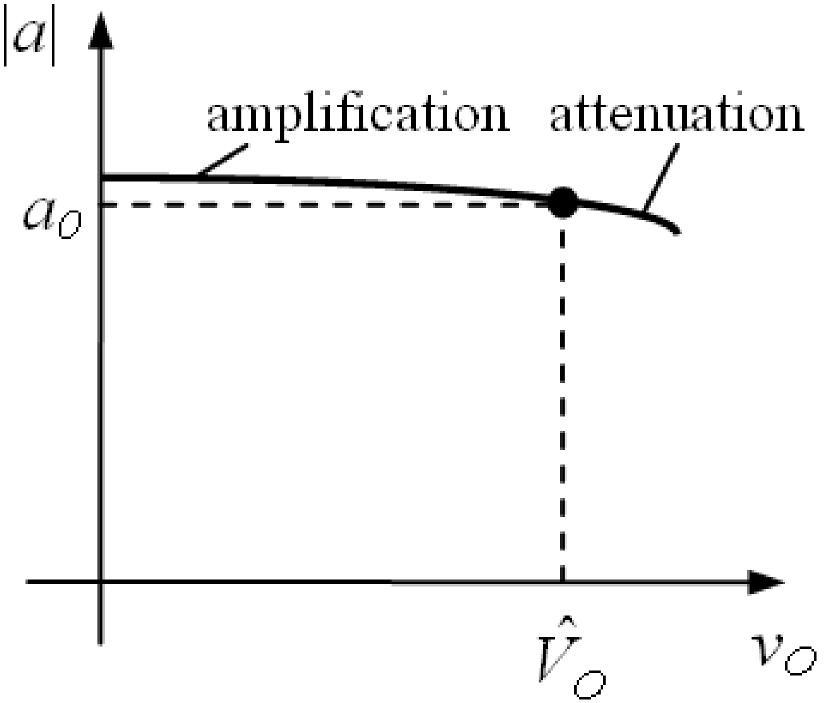
$|a(j\omega)||r(j\omega)| < 1$ oscillations are attenuated - zero

$|a(j\omega)||r(j\omega)| > 1$ oscillations are amplified - saturation

$|a(j\omega_0)||r(j\omega_0)| = 1$ oscillations are maintained - oscillate

Stability of the oscillation amplitude

Automatic gain control - depending on the output voltage magnitude $\hat{V}_o$

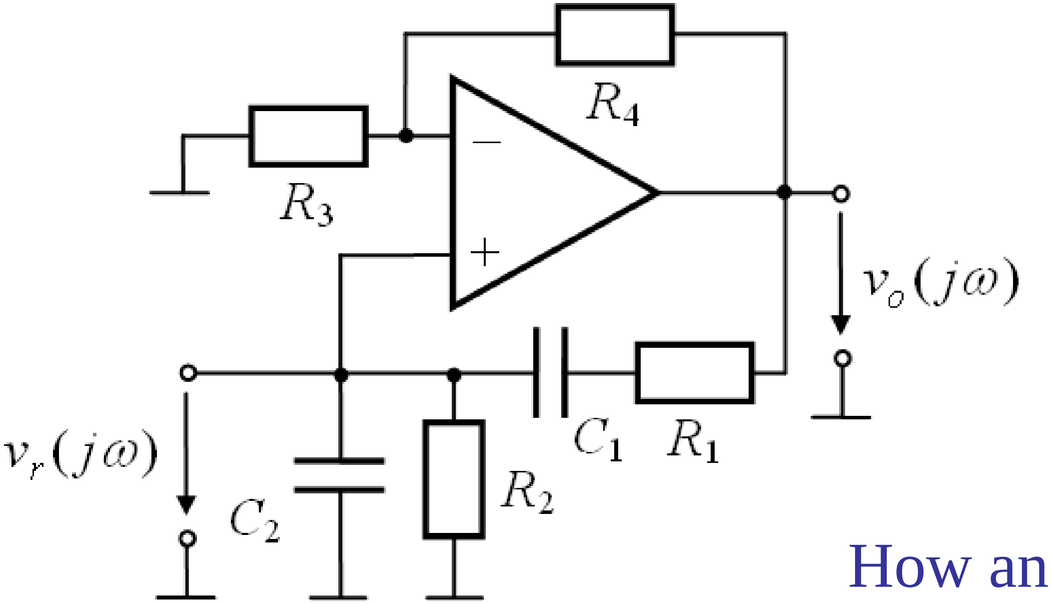


$$v_o = \hat{V}_o \sin 2\pi f_0 t$$

$$|r(j\omega)| = cst$$

$$\underline{\hat{V}_o \uparrow}, |a(j\omega_0)| \downarrow, \underline{\hat{V}_0 \downarrow}$$

AGC for WIEN bridge oscillator



$$a = 1 + \frac{R_4}{R_3}$$

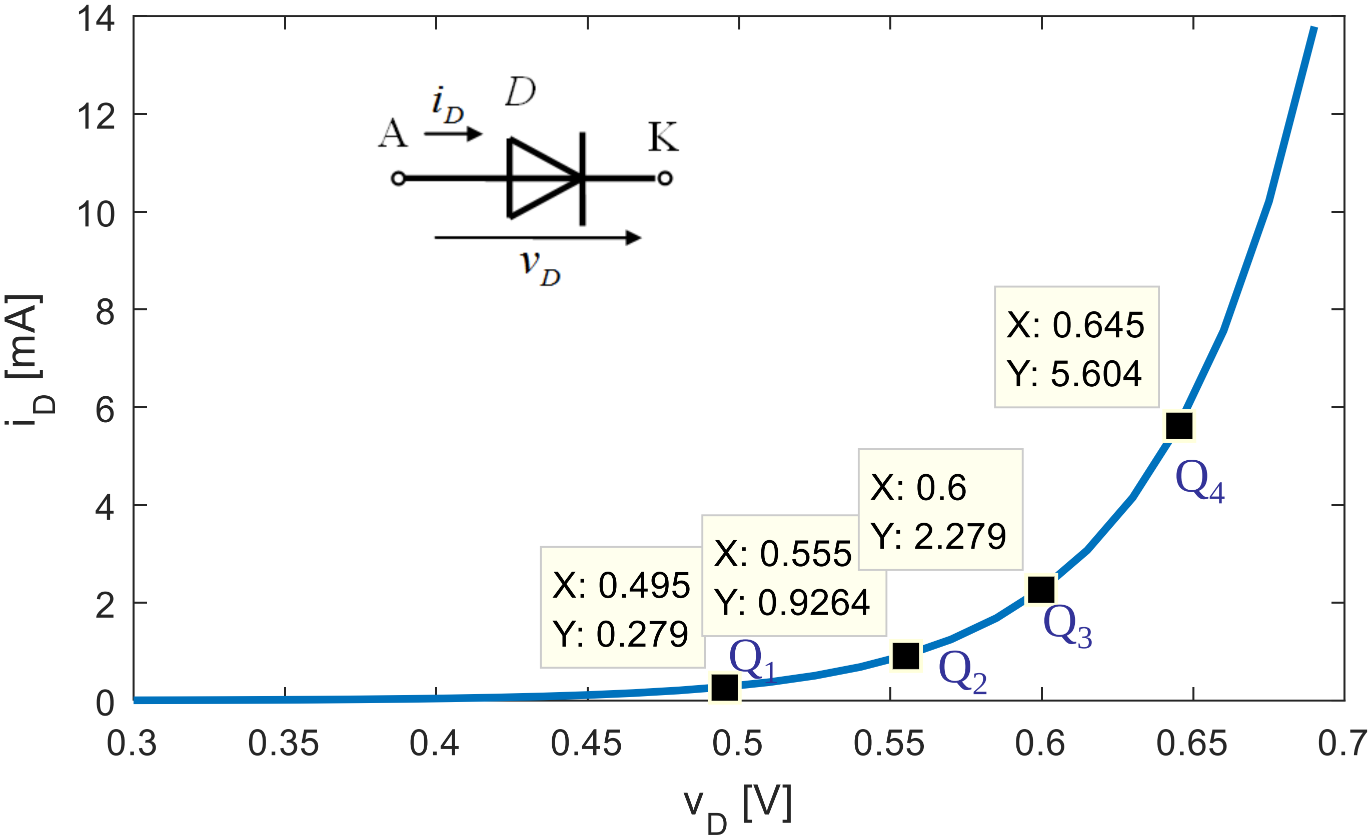
How an AGC can be implemented, so that *a* will depend on v_o value?

- R_4 - dependent on v_o

or

- R_3 - dependent on v_o

Diode revisited - as variable resistor



Static resistance of a diode in the operating point

$$r_D = \frac{V_D}{I_D}$$

$$r_{D1} = \frac{V_{D1}}{I_{D1}} = \frac{0.495\,\mathrm{V}}{0.279\,\mathrm{mA}} = 4.24\,\mathrm{k}\Omega$$

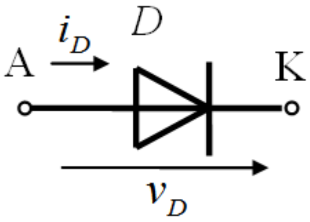
$$r_{D2} = \frac{V_{D2}}{I_{D2}} = \frac{0.555\,\mathrm{V}}{0.926\,\mathrm{mA}} = 0.599\,\mathrm{k}\Omega$$

$$r_{D3} = \frac{V_{D3}}{I_{D3}} = \frac{0.6\,\mathrm{V}}{2.279\,\mathrm{mA}} = 0.263\,\mathrm{k}\Omega$$

$$r_{D4} = \frac{V_{D4}}{I_{D4}} = \frac{0.645\,\mathrm{V}}{5.604\,\mathrm{mA}} = 0.115\,\mathrm{k}\Omega$$

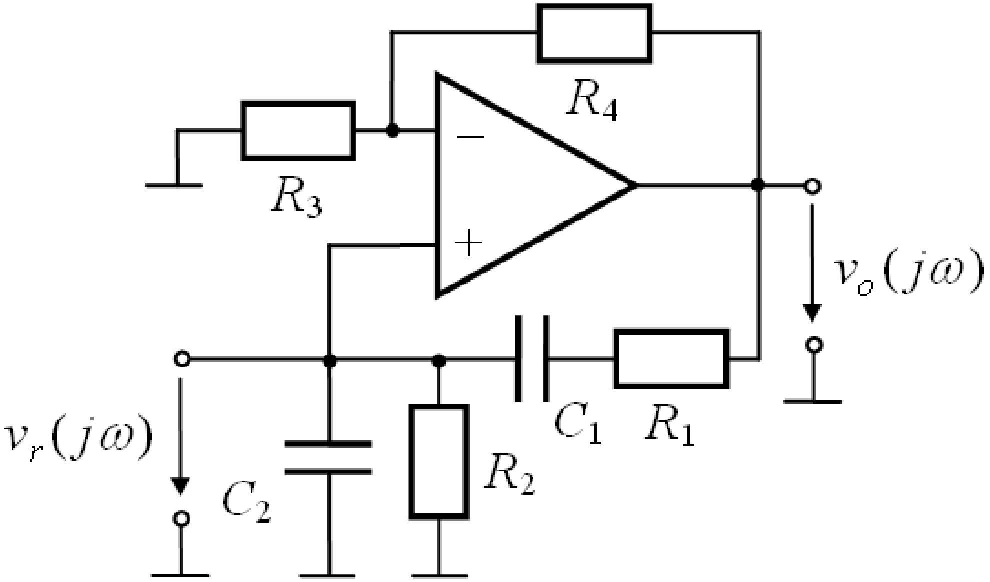
If the voltage drops, V_D **increases;** the equivalent static resistance r_D **decreases**

AGC using Diodes - how?



$$r_D = \frac{V_D}{I_D}$$

If the voltage drops, V_D increases; the equivalent static resistance r_D decreases



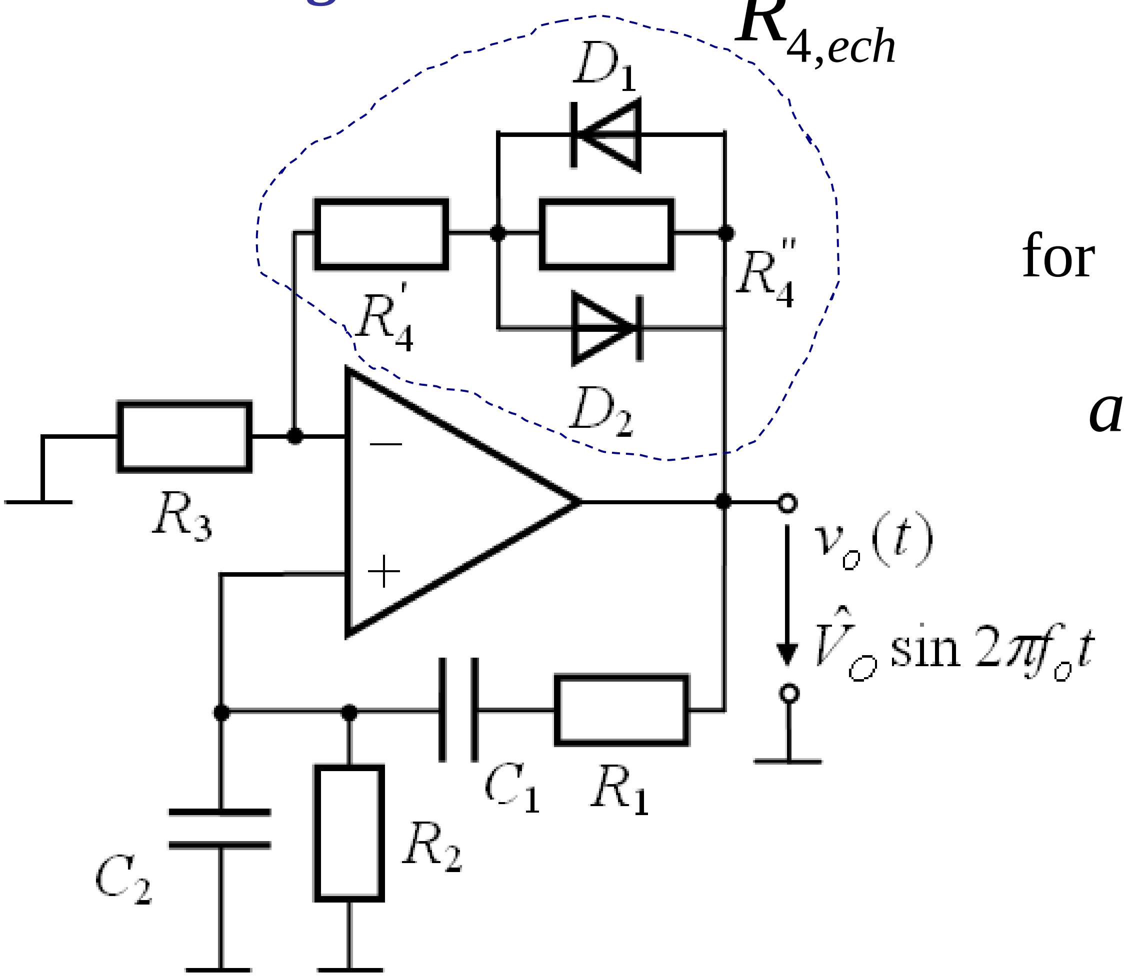
$$a = 1 + \frac{R_4}{R_3}$$

- R_4 - dependent on v_o

or

- R_3 - dependent on v_o

1. AGC using diodes



$$a = 1 + \frac{R_{4,ech}}{R_3}$$

for $v_o(t)$ small, $D_1, D_2 - (off)$

$$a_{off} = 1 + \frac{R_4' + R_4''}{R_3}$$

$$|a_{off}(j\omega_0)||r(j\omega_0)| > 1$$

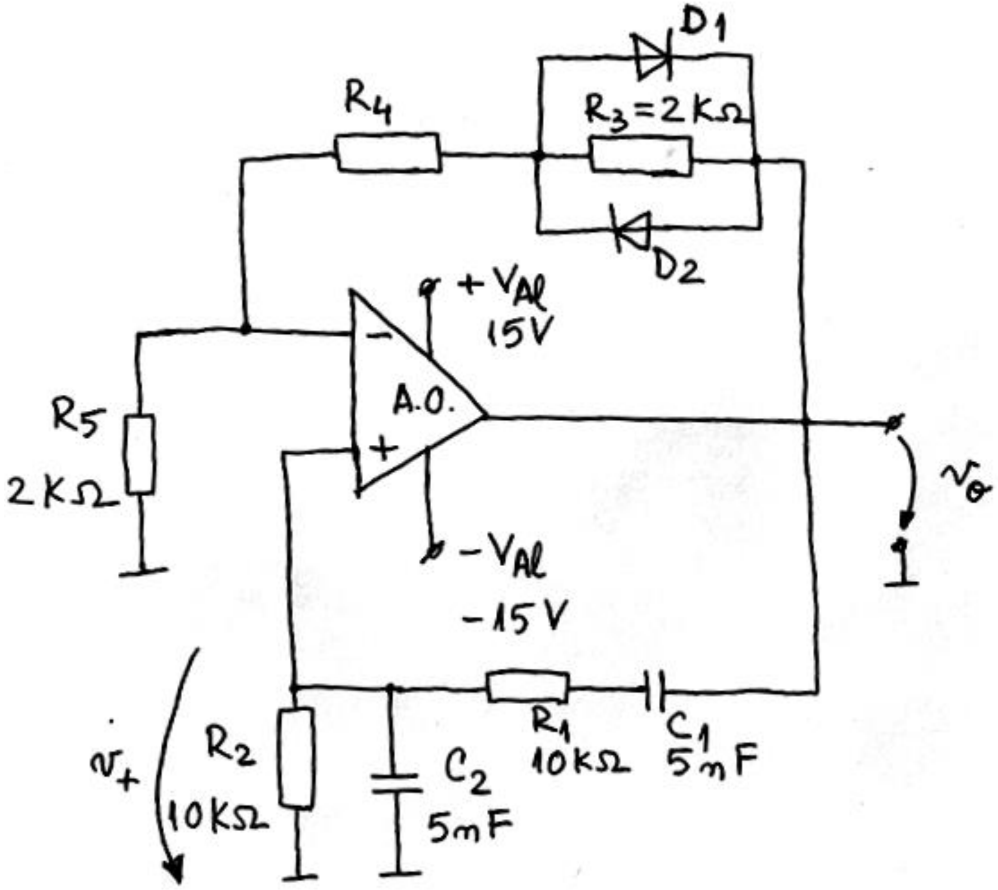
$v_o(t)$ increases,
$D_1 - (on)$ on the positive half-cycle
$D_2 - (on)$ on the negative half-cycle

$$a_{on} = 1 + \frac{R_4' + R_4'' \parallel r_D}{R_3}$$

$|a_{on}(j\omega_0)||r(j\omega_0)| = 1$ to maintain oscillations

$\hat{V}_o$ is given by the value of r_D

Problem



a) How does the voltages $v_o(t)$ and $v_+(t)$ look like in the steady-state regime? What is the oscillation frequency?

b) Size R_4 so that the circuit will maintain the oscillation. For the maximum value (magnitude) of the sinusoidal output, consider $r_{D1}= r_{D2}=$ 0.5 kΩ. Verify if the oscillation can start.

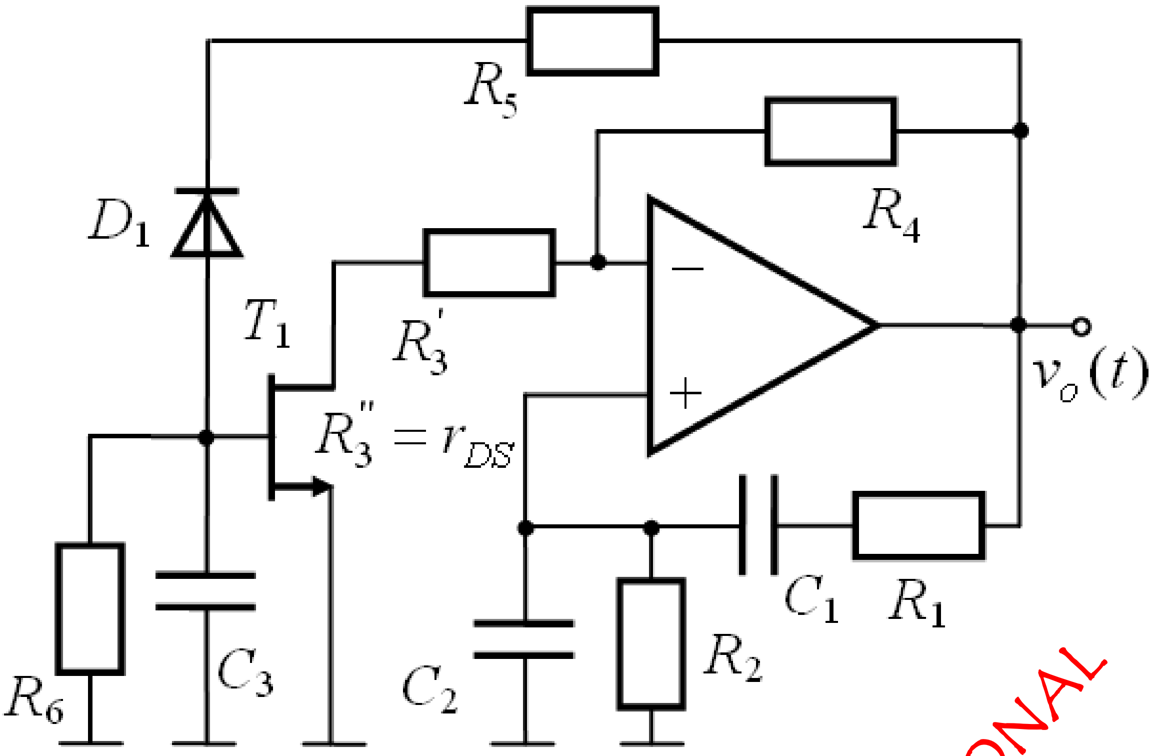
c) What is the magnitude of $v_o(t)$ in the conditions of question b), if the voltage drop across one diode is v_D = 0.58 V for the equivalent resistance r_D= 0.5 kΩ

d) How does the voltage $v_o(t)$ look like in the steady-state regime if D_2 diode is missing?

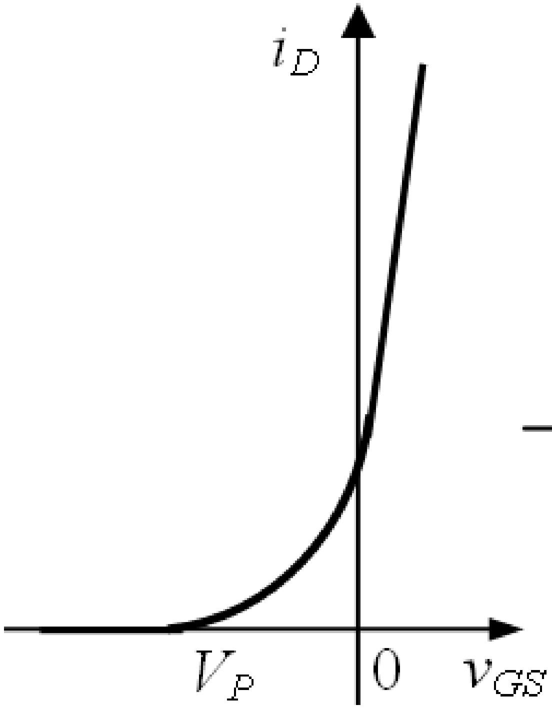
2. AGC using *n-channel depletion-type MOSFET*

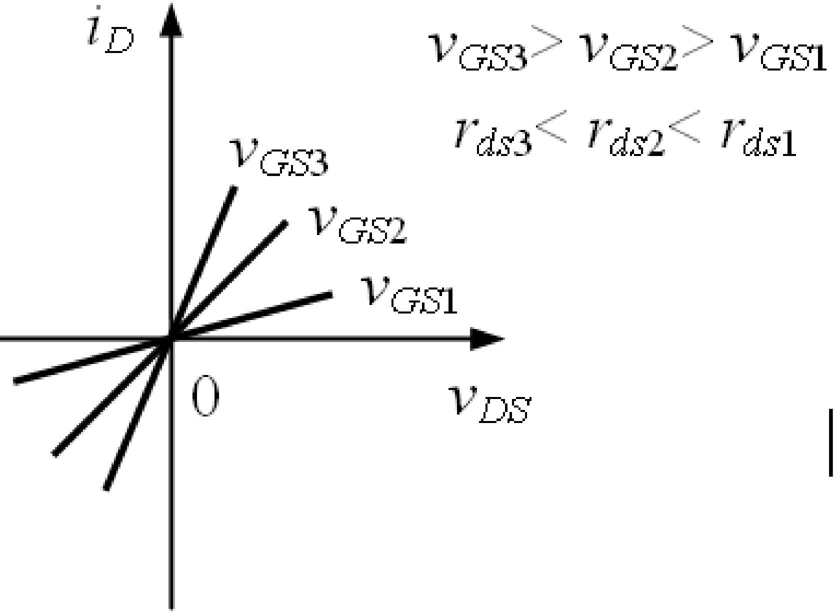
$$a = 1 + \frac{R_4}{R_3^{'} + r_{DS}}$$

$$r_{DS} \approx \frac{1}{2\beta(v_{GS} - V_{Th})}$$



OPTIONAL



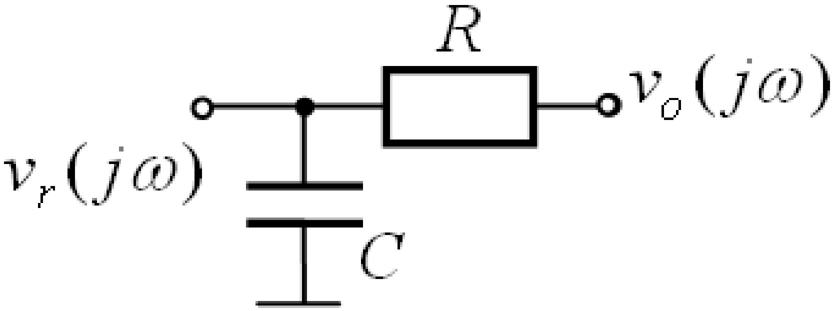


$v_{GS} < 0$

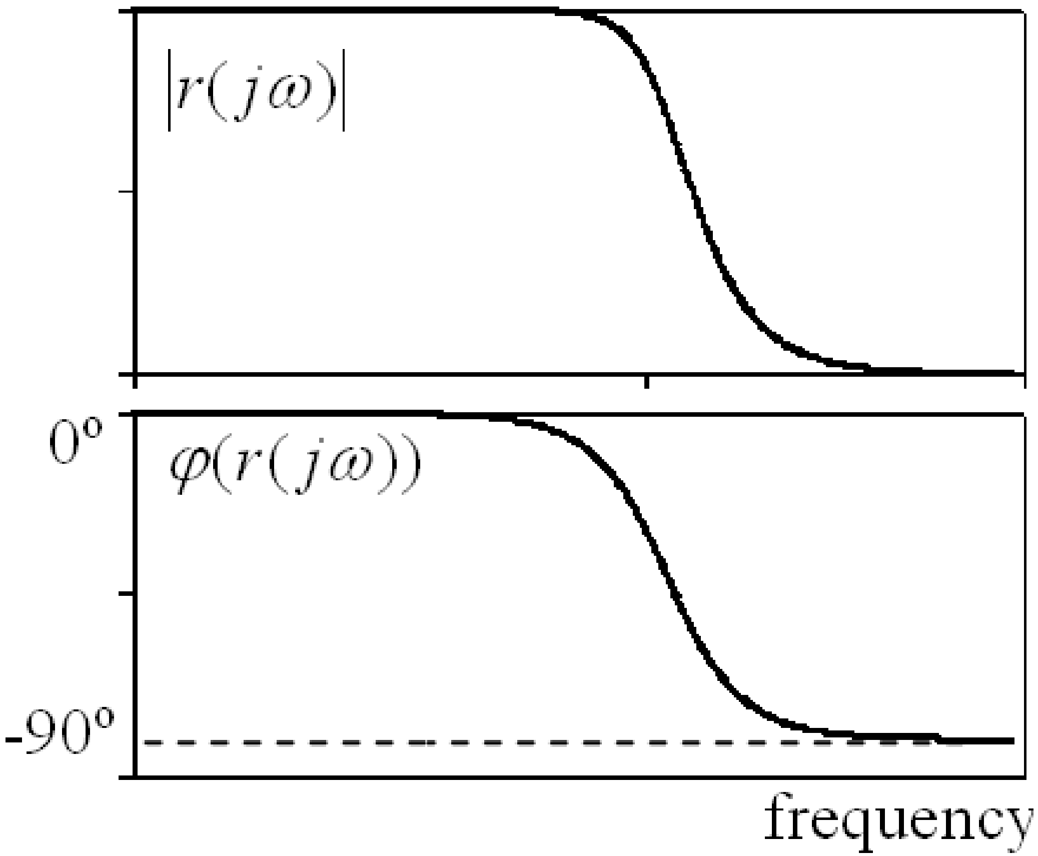
$|v_{GS}| \uparrow, r_{DS} \downarrow$

Op amp and RC ladder network oscillator

- High pass band
- Low pass band



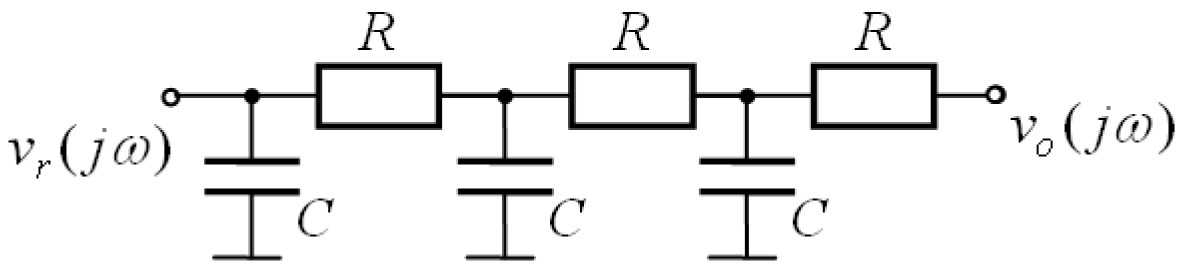
OPTIONAL



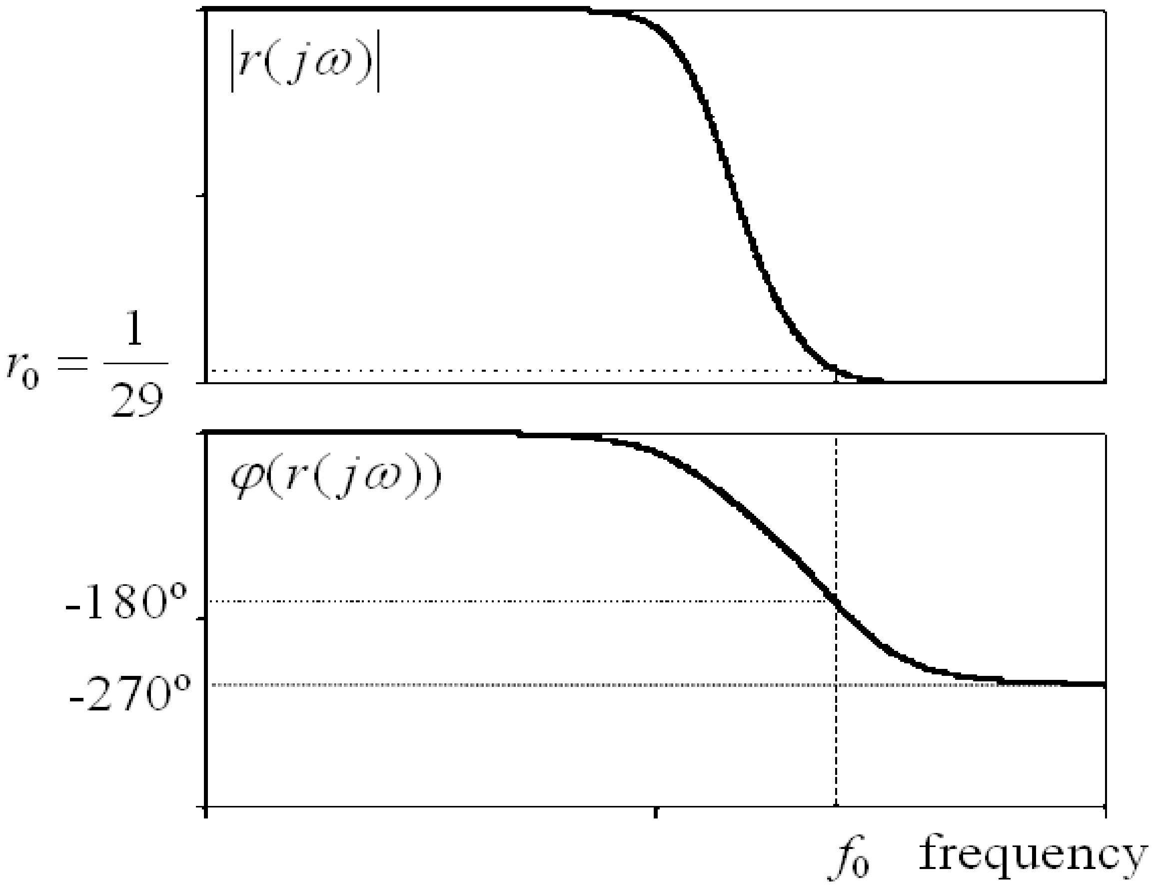
- the phase-shift is in the range of [0°; -90°]
- inverting basic amplifier
- how many identical RC cells are necessary to build an oscillator?

Low pass RC ladder with 3 cells

OPTIONAL



$$r(j\omega)=\frac{1}{1-5(\omega RC)^2+j\left[6\omega RC-(\omega RC)^3\right]}$$



$$\varphi_r=0$$

$$6\omega_0 RC-(\omega_0 RC)^3=0$$

$$\boxed{f_0=\frac{\sqrt{6}}{2\pi RC}}$$

$$\boxed{r(j\omega_0)=-\frac{1}{29}}$$

The circuit of RC ladder network oscillator

OPTIONAL