# LAB 20.  THE CONSTANT-*K* FILTERS

## 10.A. THEORY

THE LOW PASS CONSTANT-*K* FILTER

The constant-*k LPF* can have the configurations from *Figure 10.1*. The cutoff frequency is given by:

$$\omega_t = \frac{2}{\sqrt{LC}} \qquad (10.1)$$

**Figure 10.1**

Generally the filter works on a constant load *(Rs)*. To design the filter, $R_s$ and $\omega_t$ are given. The matching can not be done at any frequency therefore we have to choose the frequency at which the filter will match. Most of the times, *LPF* matches in d.c. ($\omega = 0$). The elements of the filter are given by:

$$L = \frac{2R_s}{\omega_t} \qquad C = \frac{2}{\omega_t R_s} \qquad (10.2)$$
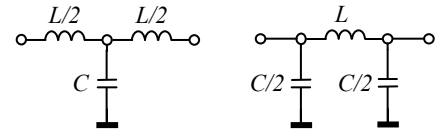
THE HIGH PASS CONSTANT-*K* FILTER

The possible configurations of the constant-*k HPF* are shown in *Figure 10.2*. The cutoff frequency is given by:

$$\omega_t = \frac{1}{2\sqrt{LC}} \qquad (10.3)$$

If we are interesting in matching at very high frequency ($\omega \to \infty$), then *L* and *C* are given by:

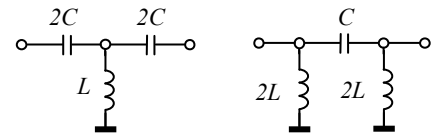$$L = \frac{R_s}{2\omega_t} \qquad C = \frac{1}{2R_s\omega_t} \qquad (10.4)$$

**Figure 10.2**

THE BAND PASS CONSTANT-k FILTER

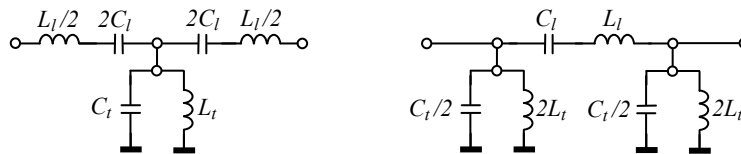The constant-*k BPF* configurations are shown in *Figure 10.3*.

**Figure 10.3**

If the cutoff frequencies ($\omega_L$ and $\omega_H$) and the load ($R_s$) are given, then the elements of the filter are given by:

$$\begin{cases} L_l = \dfrac{2R_s}{\omega_s - \omega_i} \\[3mm] C_l = \dfrac{\omega_s - \omega_i}{2R_s\omega_0^2} \end{cases} \qquad \begin{cases} L_t = \dfrac{(\omega_s - \omega_i)R}{2\omega_0^2} \\[3mm] C_t = \dfrac{2}{(\omega_s - \omega_i)R} \end{cases} \qquad (10.5)$$

THE BAND STOP CONSTANT-k FILTER

The configurations of *BEF* are given in *Figure 10.4*. If the cutoff frequencies ($\omega_L$ and $\omega_H$) and the load ($R_s$) are given, then the elements of the filter are given by:

$$\begin{cases} L_l = \dfrac{2R_s(\omega_s - \omega_i)}{\omega_0^2} \\[3mm] C_l = \dfrac{1}{2R_s(\omega_s - \omega_i)} \end{cases} \quad \begin{cases} L_t = \dfrac{R_s}{2(\omega_s - \omega_i)} \\[3mm] C_t = \dfrac{2(\omega_s - \omega_i)}{R_s\omega_0^2} \end{cases} \qquad (10.6)$$
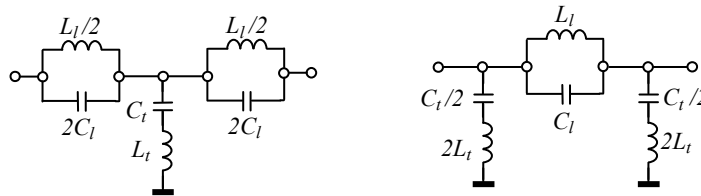


**Figure 10.4**

▶ CONNECTING THE FILTERING CELLS

The constant-*k* cells have qualities and limitations:
*Qualities:*
- ➤ they are easy to design (the configurations and the relation are very simple);
- ➤ the attenuation in the stop band tends to infinity at frequencies that are too far away from the cutoff frequencies.

*Limitations:*
- ➤ the attenuation increases very slowly in the stop band, near the pass band; therefore the stop band and the pass band are not clearly enough delimited;
- ➤ in the pass band the characteristic impedance varies a lot with frequency, so the filter is far away from being matched; as a consequence, the attenuation will not be zero in the pass band.

There are a few ways to eliminate these limitations. A way is to replace the constant-*k* filters with more complex filters (for example, the *m*-derived filters, that will be studied in the next lab).

Another way is to use more cells connected in chain. The cells must have the same characteristic impedance (in order to reduce the non-matching attenuations), therefore they will have the same cutoff frequencies.

Unfortunately, using more cells in chain won't solve the problem of correctly matching; when the attenuation increases in the stop band (desired effect), it also increases in the pass-band too (non-desired effect).

## 10.B. PROBLEMS

**10.B.1.** Design a constant-K *LPF*, the *T* configuration, with the cutoff frequency *100kHz* and the load *300Ω*.

**10.B.2.** Design a constant-K *HPF*, the *T* configuration, with the cutoff frequency *100kHz* and the load *300Ω*.

**10.B.3.** Design a constant-K *BPF*, the *T* configuration, with the cutoff frequencies *100kHz* and *200kHz,* and the load *300Ω*.

**10.B.4.** Design a constant-K *BEF*, the *T* configuration, with the cutoff frequencies *100kHz* and *200kHz,* and the load *300Ω.*.

## 10.C. LAB WORK

We'll compare the characteristics of the circuit containing one, two and three constant-k filter cells, with the schematic from *Figure 10.5*.

Set the *VPULSE* source so that it generates a rectangular signal between *±1V*, having the frequency *20kHz* and the duty-cycle *50%.* Set the parameter *AC* of the source at *2V* (at the frequency at which the filters are matched, the voltage on the load will be of *1V*).

Create a profile named "*time*", of *Time Domain* type (*Run to time = 5ms, Maximum step size = 0.5μs* and check *SKIPBP* ).

Create another profile named *"freq"*, of *AC Sweep/Noise* type (*Start frequency = 1kHz*, *End frequency = 500kHz* , with *1000* points/decade, *logarithmic* scale).
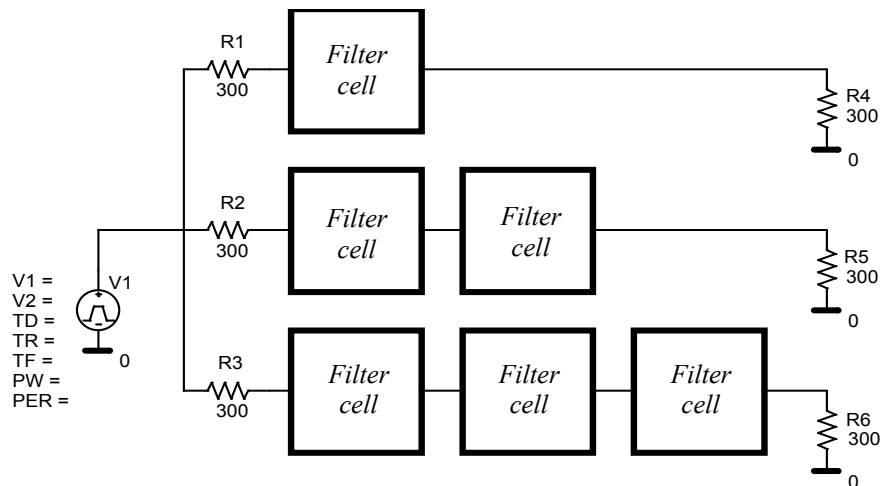


***Figure 10.5***

**10.C.1.** First we'll study the **LPF**. Each filter cell will have the configuration from *Figure 10.1.a*, and the elements have the values from problem *10.B.1*.

Make active the *"freq"* profile and run the simulation. Display the voltage on the load of the circuit with one cell (the first circuit). Notice the typically aspect of *LP* characteristic.

From *Plot → Axis Settings* modify the X-axis scale from logarithmic to linear. Measure the gain at the cutoff frequency and compare it with the theoretical value (*0.707107*).

**10.C.2.** Display on the same plot the voltages on the loads from the circuits with *2* and *3* cells.

Compare the three characteristics. Notice that the characteristics of the filters with *2* and *3* cells have oscillations in the upper side and their slopes are higher.

**10.C.3.** Make active the *"time"* profile and run the simulation. Display the voltage of the source and the voltages on the three loads. Notice the effects of using more cells: the more cells we use, the more delayed the answer is and the more it oscillates.

Push **FFT** and place the *4* signals on different plots, in the next order: the input voltage, the output voltage on the 1st, 2nd and 3rd circuit. Notice the effect of the filters on the spectrums.

**10.C.4.** Replace the *LPF* cells with **BPF** cells and repeat parts *10.C.1 – 10.C.3*.

Notice that the characteristics have a geometric symmetry with respect to the central frequency:

$$f_0 = \sqrt{f_i f_s} \qquad\qquad (10.7)$$

Measure two frequency $f_1$ and $f_2$ at which the gain has the same value (arbitrarily chosen) and verify *(10.7)*.

**10.C.5.** Find and compare the width of the band pass for each characteristic. Notice that using more cells the attenuation in the band pass increases.