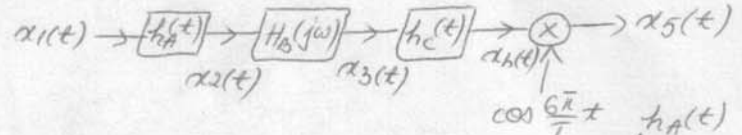
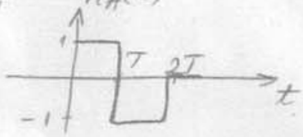
CONCURSUL PROFESIONAL
„TUDOR TĂNĂSESCU"
Secțiunea „Semnale, Circuite și Sisteme"
31 mai 2008

PROBLEMA 1 (UNIVERSITATEA „DUNĂREA DE JOS" - GALAȚI)

SE CONSIDERĂ SCHEMA BLOC DIN FIGURĂ:



$h_A(t)$



unde: - $h_A(t)$ are forma:

- $x_1(t) = \delta(t) - \delta(t-2T)$

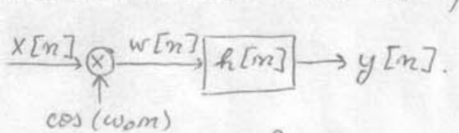
- $H_B(j\omega) = \dfrac{1}{j\omega T}$

- $h_C(t) = \dfrac{4}{T}\,\mathrm{sinc}\left(\dfrac{4\pi}{T}t\right)$

1. SĂ SE DETERMINE FORMA SEMNALULUI $x_2(t)$ ȘI SĂ SE CALCULEZE FUNCȚIA SA SPECTRALĂ
2. SĂ SE DETERMINE FORMA SEMNALULUI $x_3(t)$ ȘI SĂ SE CALCULEZE FUNCȚIA SA SPECTRALĂ, SĂ SE REPREZINTE CALITATIV FUNCȚIA DENSITĂȚII SPECTRALE DE AMPLITUDINE
3. SĂ SE REPREZINTE FUNCȚIA ~~DENSITATE~~ SPECTRALĂ A SEMNALELOR $x_4(t)$ ȘI $x_5(t)$.
4. CARE ESTE FRECVENȚA DE EȘANTIONARE MINIMĂ CU CARE POATE FI EȘANTIONAT SEMNALUL $x_5(t)$.

PROBLEMA 2 (UNIV. TEHNICĂ CLUJ-NAPOCA)

Se consideră schema-bloc din figură:



unde: $x[n] = \delta[n] + \dfrac{2\omega_0}{\pi(2\omega_0 n - 3\pi)}$

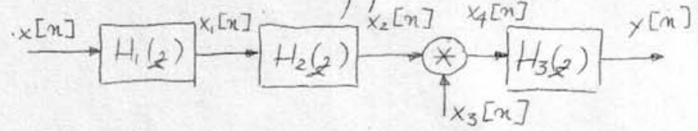
$$h[n] = \cos\left(\frac{\omega_0 n}{3}\right) + \alpha\cdot\cos\left(\frac{\omega_0 n}{3} + \frac{\pi}{4}\right) \quad ,|\omega_0| < \pi,\ \alpha \in \mathbb{R}\setminus\{0\}$$

(a) Să se determine și să se reprezinte grafic modulul și argumentul transformatei Fourier a semnalului $w[n]$.

(b) Știind că $y[n] = \sqrt{10}\cdot\cos\left(\frac{\omega_0 n}{3} + \beta\right)$, $\beta \in \mathbb{R}\setminus\{0\}$, să se determine α și β.

UNIVERSITATEA POLITEHNICA BUCUREȘTI

3. Se dă schema din figură



Blocurile $H_1(z)$, $H_2(z)$ și $H_3(z)$ sunt trei SDLIT, având funcțiile de transfer:

$$H_1(z) = 1 + z^{-1} \qquad H_2(z) = z^{-1} - z^{-4} \qquad H_3(z) = \sum_{k=0}^{NM-1} h_k z^{-k} \quad N, M \in \mathbb{N}^*$$

unde $h_k = \begin{cases} (-1)^p \dfrac{N!}{p!(N-p)!}, & k = Mp \\ 0, & k \neq Mp \end{cases}$

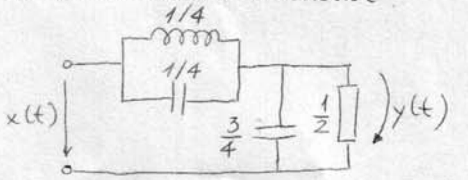
Se dau: $x[n] = \delta[n]$, $x_3[n] = \sum_{k=-\infty}^{\infty} \delta[n-6k]$

a) Reprezentați semnalele $x_1[n]$, $x_2[n]$ și $x_4[n]$ în timp și spectrele de amplitudini corespunzătoare.

b) Determinați valoarea minimă a lui M a.î. $y[n] = 0$

UNIVERSITATEA POLITEHNICA BUCUREȘTI

4. Se consideră circuitul.



a) Să se calculeze funcția de transfer $H(s) = \dfrac{Y(s)}{X(s)}$ și funcția pondere $h(t) = \mathcal{L}^{-1}\{H(s)\}$.

Reprezentați grafic $h(t)$.

b) Să se calculeze răspunsul $y_1(t)$ la semnalul $x_1(t) = \dfrac{2}{3}\cos t \cos 3t$ și să se reprezinte grafic.

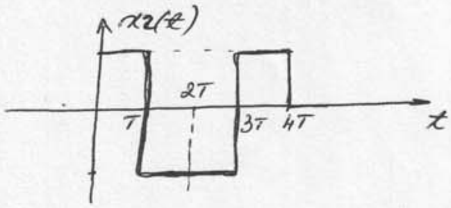
c) Să se calculeze răspunsul $y_2(t)$ la semnalul $x_2(t) = \sin 4t \, u(t)$ și să se reprezinte grafic.

($u(t)$ este funcția treaptă unitate)

REZOLVARE ŞI BAREM - PROBLEMA 1

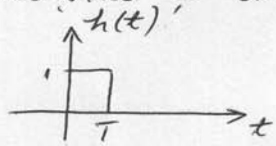
oficiu 1p

(1) $x_2(t) = x_1(t) * h_A(t) = h_A(t) - h_A(t-2T)$



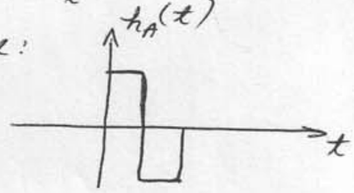
1p

Pentru a determina forma spectrală a semnalului $h_A(t)$ plecăm de la semnalul a cărui densitate spectrală o cunoaştem şi anume $h(t)$:



$$\Rightarrow H(\omega) = T\cdot e^{-j\frac{\omega T}{2}}\,\mathrm{sinc}\frac{\omega T}{2}$$

Prin urmare:



$$\Rightarrow H_A(\omega) = Te^{-j\frac{\omega T}{2}}\,\mathrm{sinc}\frac{\omega T}{2}\left(1-e^{-j\omega T}\right);$$

$$\Rightarrow H_A(\omega) = Te^{-j\frac{\omega T}{2}}\,\mathrm{sinc}\frac{\omega T}{2}\cdot e^{-j\frac{\omega T}{2}}\cdot\frac{e^{j\frac{\omega T}{2}}-e^{-j\frac{\omega T}{2}}}{2j\frac{\omega T}{2}}\cdot 2j\cdot\frac{\omega T}{2} =$$

$$= T\cdot e^{-j\frac{\omega T}{2}}\cdot\mathrm{sinc}\frac{\omega T}{2}\cdot e^{-j\frac{\omega T}{2}}\cdot\mathrm{sinc}\frac{\omega T}{2}\cdot(j\omega T) = j\omega T^2 e^{-j\omega T}\,\mathrm{sinc}^2\frac{\omega T}{2}$$

$$\boxed{H_A(\omega) = j\omega T^2 e^{-j\omega T}\,\mathrm{sinc}^2\frac{\omega T}{2}}$$

1p

$x_2(t) = h_A(t) - h_A(t-2T)$

$$X_2(\omega) = H_A(j\omega)\left(1-e^{-j2\omega T}\right) = H_A(j\omega)\cdot e^{-j\omega T}\cdot\left(e^{j\omega T}-e^{-j\omega T}\right)\cdot\frac{2j}{2j} =$$

$$= 2jH_A(j\omega)e^{-j\omega T}\sin\omega T$$

$$\boxed{X_2(\omega) = -2\omega T^2 e^{-j2\omega T}\,\mathrm{sinc}^2\frac{\omega T}{2}\,\sin\omega T}$$

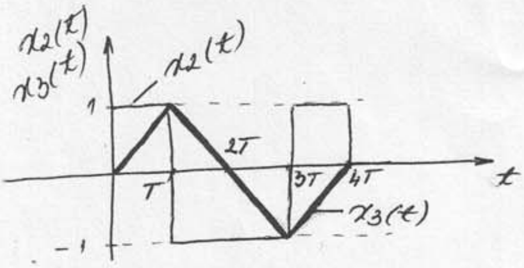
1p

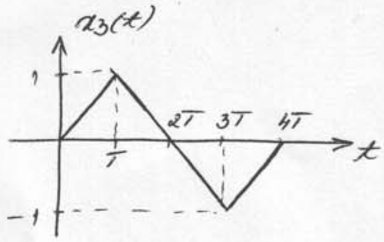
(2) $H_B(j\omega) = \frac{1}{j\omega T}$ - este răspunsul la frecvenţă al unui integrator de ecuaţie: $x_3(t) = \frac{1}{T}\int_0^T x_2(\tau)d\tau$

$$X_3(\omega) = H_B(j\omega)X_2(\omega) = \frac{1}{j\omega T}X_2(\omega) = 2jTe^{-j2\omega T}\,\mathrm{sinc}^2\frac{\omega T}{2}\sin\omega T$$

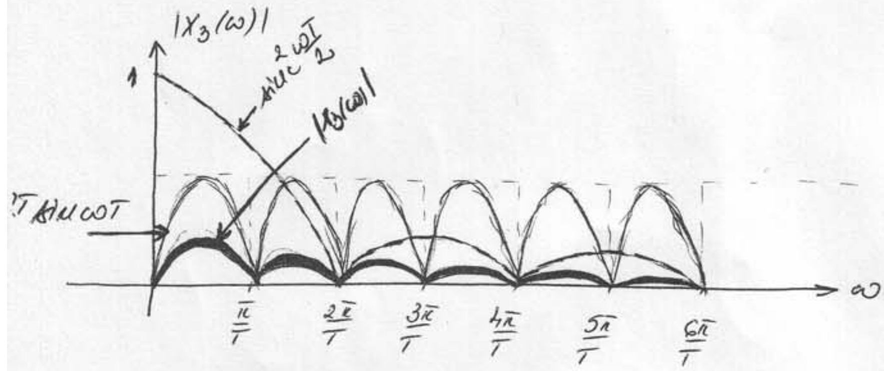
$$|X_3(\omega)| = |2T\sin\omega T|\,\mathrm{sinc}^2\frac{\omega T}{2}$$

1p





1p

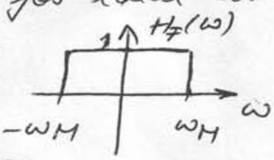


1p

③ (!) $h_c(t) = \frac{4}{T}\,\mathrm{sinc}\left(\frac{4\pi}{T}t\right)$

Funcția pondere a filtrului trece jos ideal este:

(!!) $h_{T}(t) = \frac{\omega_H}{\pi}\,\mathrm{sinc}\,\omega_H \cdot t$



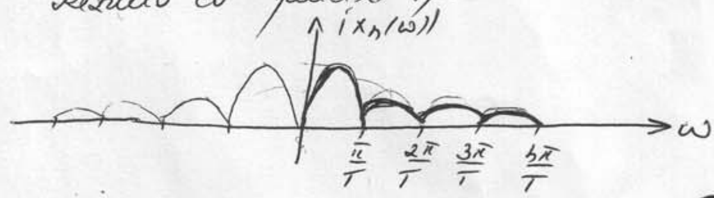
1p

Comparând (!) și (!!) rezultă

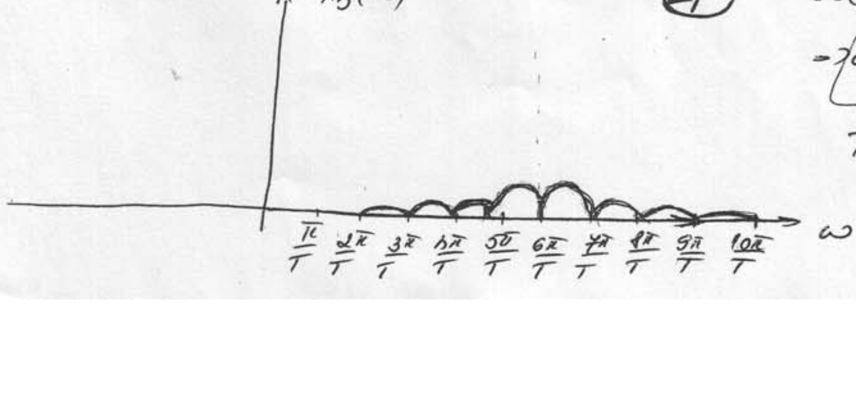
$$\omega_H = \frac{4\pi}{T} \Rightarrow h_T(t) = \frac{\frac{4\pi}{T}}{\pi}\,\mathrm{sinc}\left(\frac{4\pi}{T}t\right) = \frac{4}{T}\,\mathrm{sinc}\left(\frac{4\pi}{T}\cdot t\right)$$

$\Rightarrow h_c(t) = h_T(t)$

Rezultă că funcția spectrală a semnalului $x_4(t)$ este



1p



④

$\omega_{max} = \frac{10\pi}{T}$

$\omega_e = 2\cdot\frac{10\pi}{T} \Rightarrow$

$\Rightarrow \boxed{\omega_e = 10\omega \text{ minimă}}$ 1p

$T_e = \frac{T}{10}$

REZOLVARE ȘI BAREM – PROBLEMA 2

of. (1p)

(a) $w[n] = x[n]\cdot\cos(\omega_0 n) =$

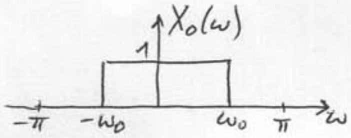
$$= \delta[n]\cdot\cos(\omega_0 n) + \frac{2\omega_0}{\pi(2\omega_0 n - 3\pi)}\cdot\cos(\omega_0 n) =$$

$$= \delta[n]\cdot\cos(0) + \frac{\omega_0\cdot\cos(\omega_0 n)}{\pi\left(\omega_0 n - \frac{3\pi}{2}\right)} =$$

$$= \delta[n] + \frac{\omega_0}{\pi}\cdot\frac{\sin\left(\omega_0 n - \frac{3\pi}{2}\right)}{\omega_0 n - \frac{3\pi}{2}} =$$

$$= \delta[n] + \frac{\omega_0}{\pi}\cdot Sa\left(\omega_0 n - \frac{3\pi}{2}\right)$$ (1p)

Fie $x_0[n] = \frac{\omega_0}{\pi}\cdot Sa(\omega_0 n) \Rightarrow X_0(\omega) = \begin{cases} 1, & \text{pt. } |\omega|<\omega_0 \\ 0, & \text{pt. } \omega_0<|\omega|<\pi \end{cases}$ (Fig. 1)



= Fig 1 =

Rezultă: $w[n] = \delta[n] + x_0\left[n - \frac{3\pi}{2\omega_0}\right]$, deci în frecvență vom avea: $W(\omega) = 1 + X_0(\omega)\cdot e^{-j\frac{3\pi}{2\omega_0}\cdot\omega} =$

$$= \begin{cases} 1 + e^{-j\frac{3\pi}{2\omega_0}\cdot\omega}, & |\omega|<\omega_0 \\ 1, & \omega_0<|\omega|<\pi \end{cases}$$

$$1 + e^{-j\frac{3\pi}{2\omega_0}\cdot\omega} = e^{-j\frac{3\pi}{4\omega_0}\cdot\omega}\left(e^{j\frac{3\pi}{4\omega_0}\cdot\omega} + e^{-j\frac{3\pi}{4\omega_0}\cdot\omega}\right) =$$ (1p)

$$= 2\cos\left(\frac{3\pi}{4\omega_0}\cdot\omega\right)\cdot e^{-j\frac{3\pi}{4\omega_0}\cdot\omega}$$

Modulul transformatei Fourier a semnalului $w[n]$:

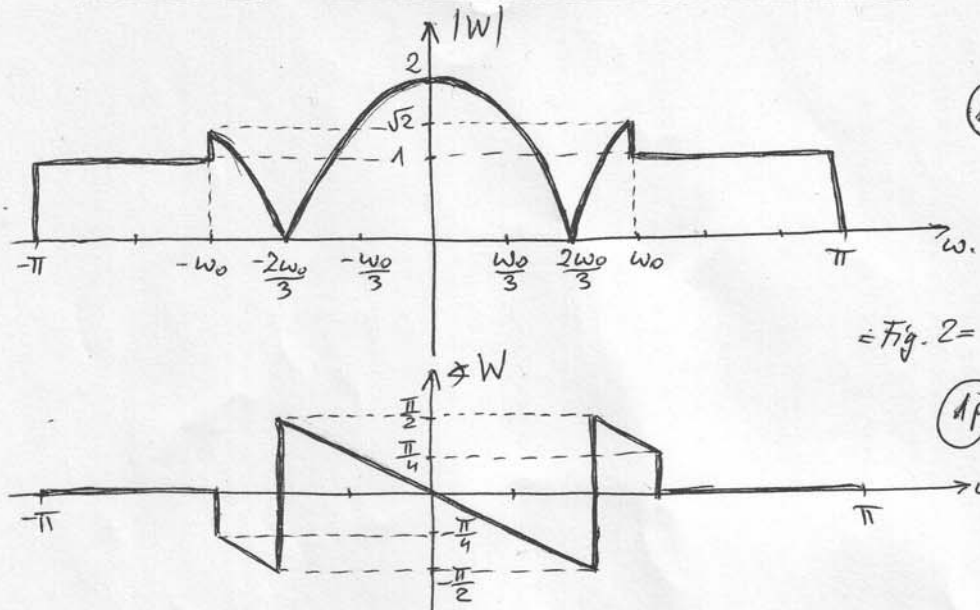
$$|W(\omega)| = \begin{cases} 2\left|\cos\frac{3\pi}{4\omega_0}\cdot\omega\right|, & |\omega|<\omega_0 \\ 1, & \omega_0<|\omega|<\pi \end{cases}$$ (1p)

Argumentul transf. Fourier:

$$\measuredangle W(\omega) = \begin{cases} -\frac{3\pi}{4\omega_0}\omega + \begin{cases} 0, & \text{dacă } \cos\left(\frac{3\pi}{4\omega_0}\omega\right)\geq 0 \\ \pi, & \text{dacă } \cos\left(\frac{3\pi}{4\omega_0}\omega\right)>0 \end{cases}, & |\omega|<\omega_0. \\ 0, & \omega_0<|\omega|<\pi \end{cases}$$

$$\measuredangle W(\omega) = \begin{cases} -\frac{3\pi}{4\omega_0}\omega, & |\omega|<\frac{2\omega_0}{3} \\ -\frac{3\pi}{4\omega_0}\omega + \pi, & \frac{2\omega_0}{3}<|\omega|<\omega_0 \\ 0, & \omega_0<|\omega|<\pi \end{cases}$$ (Fig. 2) (1p)

REZOLVARE ȘI BAREM – PROBLEMA 2 (continuare)



(1p)

= Fig. 2 =

(1p)

(b) $h[n] = \cos\left(\frac{w_0 n}{3}\right) + \alpha\cdot\cos\left(\frac{w_0 n}{3} + \frac{\pi}{4}\right) = H_0\cdot\cos\left(\frac{w_0 n}{3} + \varphi_h\right)$

$$\cos\left(\frac{w_0 n}{3}\right) + \alpha\cdot\cos\left(\frac{w_0 n}{3}\right)\frac{1}{\sqrt{2}} - \alpha\cdot\sin\left(\frac{w_0 n}{3}\right)\frac{1}{\sqrt{2}} = H_0\cdot\cos\left(\frac{w_0 n}{3}\right)\cos\varphi_h - H_0\cdot\sin\left(\frac{w_0 n}{3}\right)\sin\varphi_h$$

$$\Rightarrow \begin{cases} H_0\cdot\cos\varphi_h = 1 + \frac{\alpha}{\sqrt{2}} \\ H_0\cdot\sin\varphi_h = \frac{\alpha}{\sqrt{2}} \end{cases} \Rightarrow \begin{cases} H_0 = \sqrt{\alpha^2 + \alpha\sqrt{2} + 1} \\ \varphi_h = \operatorname{arctg}\frac{\alpha}{\alpha + \sqrt{2}} \end{cases}$$ (1p)

$$\xrightarrow{w[n]} \boxed{h[n]} \rightarrow y[n] \iff \xrightarrow{h[n]} \boxed{w[n]} \rightarrow y[n]$$ = Fig 3 =

Dacă ținem cont de echivalența din Fig. 3, deoarece $h[n]$ este un cosinus, conform metodei armonice, $y[n]$ va fi de forma:

$$y[n] = Y_0\cdot\cos\left(\frac{w_0 n}{3} + \varphi_y\right),$$

unde: $\begin{cases} Y_0 = H_0\cdot\left|W\left(\frac{w_0 n}{3}\right)\right| = \sqrt{2(\alpha^2 + \sqrt{2}\alpha + 1)} \\ \varphi_y = \varphi_h + \measuredangle W\left(\frac{w_0 n}{3}\right) = \operatorname{arctg}\frac{\alpha}{\alpha + \sqrt{2}} - \frac{\pi}{4}. \end{cases}$ (1p)

$Y_0 = \sqrt{10} \Rightarrow \alpha_1 = -2\sqrt{2}$ și $\alpha_2 = \sqrt{2}$

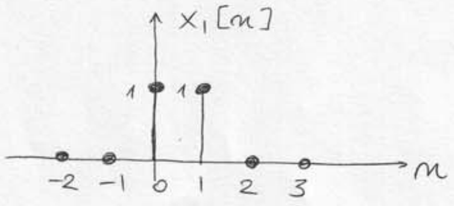
$\beta = \operatorname{arctg}\frac{\alpha}{\alpha + \sqrt{2}} - \frac{\pi}{4} \Rightarrow \begin{cases} \text{dacă } \alpha_1 = -2\sqrt{2} \Rightarrow \beta_1 = \operatorname{arctg} 2 - \frac{\pi}{4}. \\ \text{dacă } \alpha_2 = \sqrt{2} \Rightarrow \beta_2 = \operatorname{arctg}\frac{1}{2} - \frac{\pi}{4}. \end{cases}$ (1p)

— o —

Problema 3 - Barem rezolvare

a) $x_1[n] = x[n] * h_1[n]$

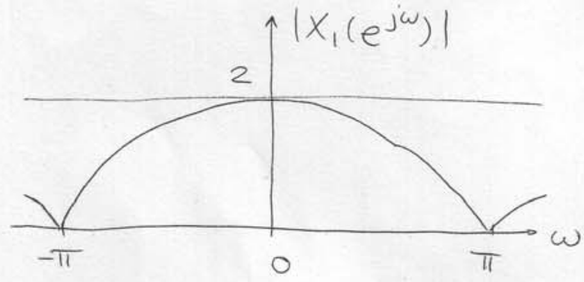
$x_1[n] = x[n] + x[n-1]$ 0.25p



0.25p

$X_1(e^{j\omega}) = 1 + e^{-j\omega} = e^{-j\frac{\omega}{2}} \cdot 2\cos\frac{\omega}{2}$ 0.25p

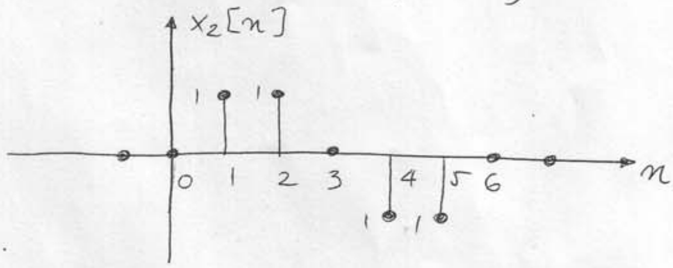
$|X_1(e^{j\omega})| = 2\left|\cos\frac{\omega}{2}\right|$ 0.5p



0.5p

$x_2[n] = x_1[n] * h_2[n]$

$x_2[n] = x_1[n-1] - x_1[n-4]$ 0.25p



0.25p

$X_2(e^{j\omega}) = X_1(e^{j\omega})e^{-j\omega} - X_1(e^{j\omega})e^{-j4\omega} =$

$= e^{-j\omega}X_1(e^{j\omega})\,(1 - e^{-j3\omega}) =$

$= e^{-j\omega}X_1(e^{j\omega}) \cdot e^{-j\frac{3}{2}\omega} \cdot 2j\sin\frac{3\omega}{2}$ 0.25p

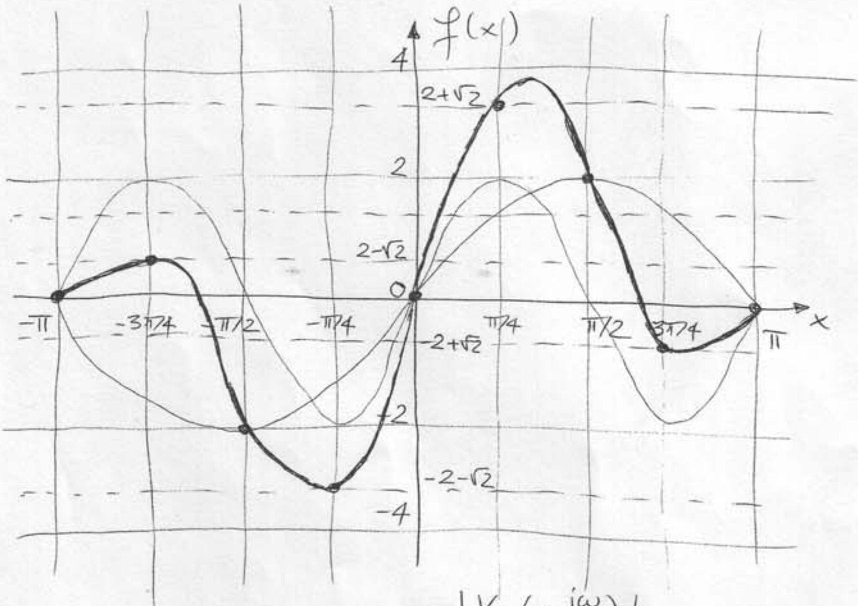
$$X_2(e^{j\omega}) = 2j\, e^{-j\frac{5}{2}\omega} \cdot e^{-j\frac{\omega}{2}} \cdot 2\cos\frac{\omega}{2} \cdot \sin\frac{3\omega}{2}$$

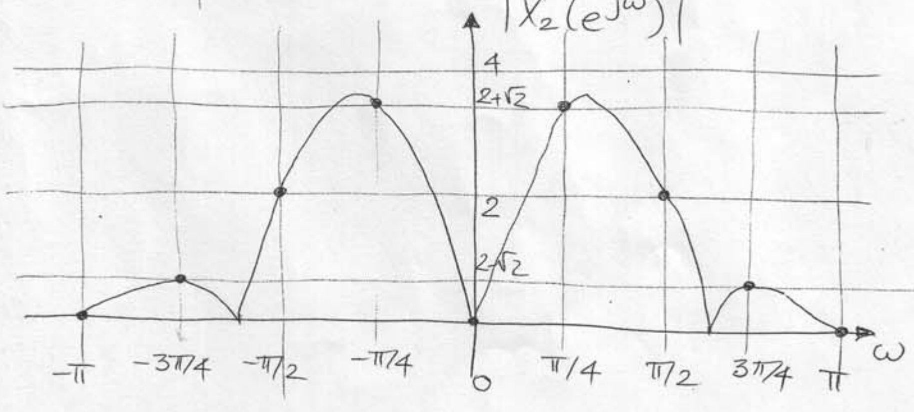
$$X_2(e^{j\omega}) = 4\cos\frac{\omega}{2} \cdot \sin\frac{3\omega}{2}\; e^{j\left(\frac{\pi}{2}-3\omega\right)}$$

$$|X_2(e^{j\omega})| = 4\left|\cos\frac{\omega}{2}\sin\frac{3\omega}{2}\right| = 2\left|\sin\omega + \sin 2\omega\right|$$ (0.5p)

Considerăm funcția $f(x) = 2\sin x + 2\sin 2x$

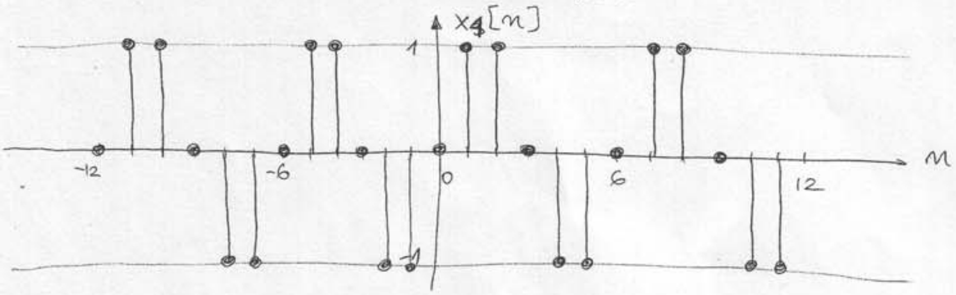
x	$-\pi$	$-3\pi/4$	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$f(x)$	0	$+(2\pm\sqrt{2})$	-2	$-(2+\sqrt{2})$	0	$2+\sqrt{2}$	2	$-2+\sqrt{2}$	0





(1p)

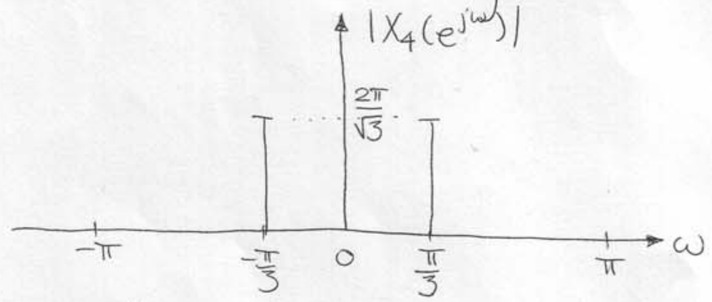
$$x_4[n] = x_2[n] * \sum_{k=-\infty}^{\infty} \delta[n-6k] = \sum_{k=-\infty}^{\infty} x_3[n-6k]$$ 0.5p



0.5p

Se observă că $x_4[n] = \frac{2}{\sqrt{3}} \sin\left(\frac{n\pi}{3}\right)$ 0.5p

Pentru $\omega \in [-\pi, \pi)$, $X_4(e^{j\omega}) = \frac{2\pi}{\sqrt{3}j}\delta(\omega - \frac{\pi}{3}) - \frac{2\pi}{\sqrt{3}j}\delta(\omega + \frac{\pi}{3})$ 0.25p



0.25p

b) $$H_3(z) = \sum_{p=0}^{N} (-1)^p \frac{N!}{p!(N-p)!} z^{-pM} =$$

$$= \sum_{p=0}^{N} (-1)^p C_N^p z^{-pM} = (1 - z^{-M})^N$$ 1p

$$H_3(e^{j\omega}) = (1 - e^{-jM\omega})^N = e^{-j\frac{MN\omega}{2}}\left(2j \sin\frac{M\omega}{2}\right)^N$$ 1p

Pentru ca $y[n] = 0 \Rightarrow H_3(e^{j\frac{\pi}{3}}) = 0$ 0.5p

$$\Rightarrow \sin\frac{M\pi}{6} = 0 \Rightarrow \frac{M\pi}{6} = k\pi \Rightarrow M = 6k,\ k \in \mathbb{Z}$$

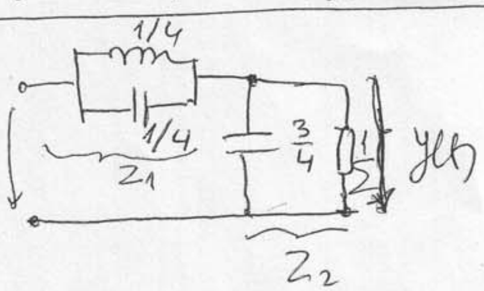
Valoarea minimă nenulă: $\boxed{M = 6}$ 0.5p

17

Problema 4 - rezolvare și barem

of 1p

a)



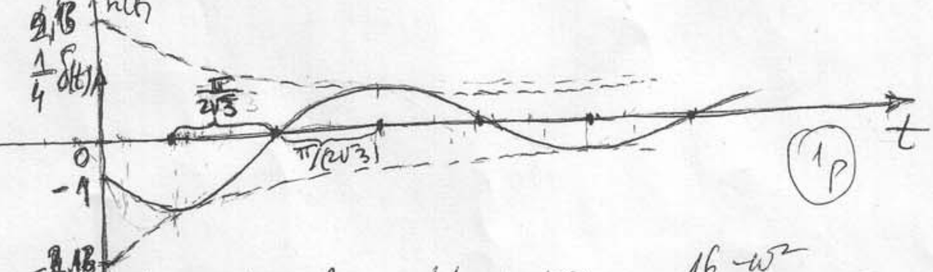
$$Z_1(s) = \frac{4s}{s^2+16};\qquad Z_2(s) = \frac{4}{3s+8}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{s^2+16}{4(s^2+2s+4)}$$ 1p

$$H(s) = \frac{1}{4} + \frac{6-s}{2(s^2+2s+4)} = \frac{1}{4} - \frac{s+1}{2[(s+1)^2+(\sqrt{3})^2]} + \frac{7}{\sqrt{3}}\frac{\sqrt{3}}{2[(s+1)^2+(\sqrt{3})^2]}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \frac{1}{4}\delta(t) - \frac{1}{2}e^{-t}\cos\sqrt{3}t\,u(t) + \frac{7}{2\sqrt{3}}e^{-t}\sin\sqrt{3}t\,u(t) =$$

$$= \frac{1}{4}\delta(t) + \sqrt{\frac{13}{3}}\cos(\sqrt{3}t - 103{,}9^\circ)\,e^{-t}u(t)$$ 2p

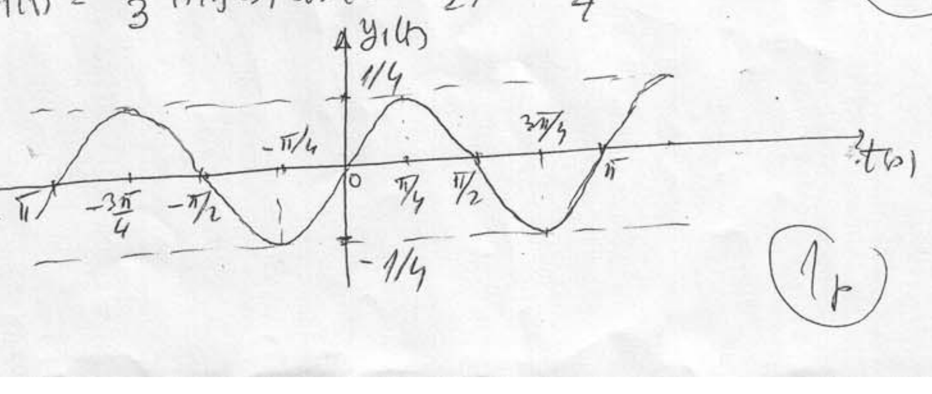


1p

b) $x_1(t) = \frac{1}{3}\cos 2t + \frac{1}{3}\cos 4t$; $H(j\omega) = \frac{16-\omega^2}{4(-\omega^2+2j\omega+4)}$

$H(j4) = 0$, $H(j2) = \frac{12}{16j} = \frac{3}{4}(-j) = \frac{3}{4}e^{-j\pi/2}$

$y_1(t) = \frac{1}{3}|H(j2)|\cos(2t - \frac{\pi}{2}) = \frac{1}{4}\sin 2t$ 1,5p



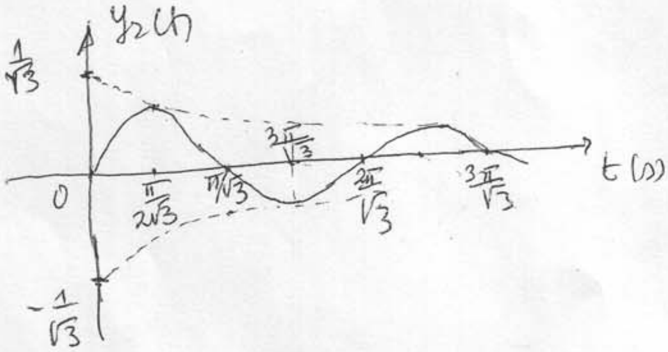
1p

c) $X_2(s) = \mathcal{L}\{x_2(t)\} = \frac{4}{s^2+16}$, $Re\{s\} > 0$

$$Y_2(s) = X_2(s)\cdot H(s) = \frac{1}{s^2+2s+4} = \frac{1}{(s+1)^2+(\sqrt{3})^2} =$$

$$= \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{(s+1)^2+(\sqrt{3})^2} \quad , \quad Re\{s\} > -1$$

$$y_2(t) = \frac{1}{\sqrt{3}} e^{-t} \sin\sqrt{3}t \, u(t)$$ 1,5p



1p